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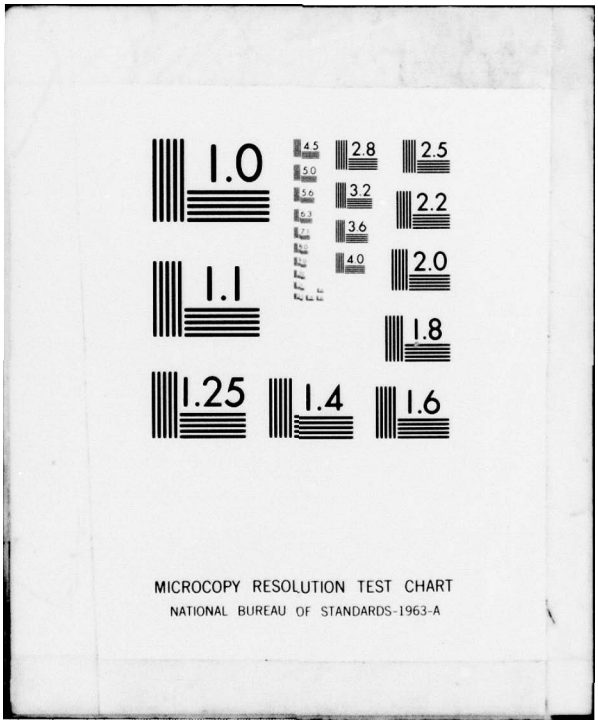
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AFAPL-TR-79-2088

ANALYSIS OF SOLIDIFICATION IN A CYLINDRICAL ANNULUS WITH INTERNAL FINS

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Solutions are presented for the one-dimensional and two-dimensional inward solidification of a material contained in a cylindrical annulus. The outer boundary is insulated and the inner boundary is subject to a constant efflux via a heat pipe. Results are presented for the axisymmetric case and for the cases of three and six longitudinal fins of rectangular cross section. The fins extend radially from inner to outer radius and were symmetrically → over

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20. Abstract

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LIST OF ILLUSTRATIONS (continued)

NOMENCLATURE

B	Fin thickness, ft
c	Specific heat of the salt, BTU/lbm^{o}F
f	Volume ratio of solid salt to cannister
G	Fin length, ft
h_{s_ℓ}	Latent heat of fusion of salt, BTU/lbm
h	Specific enthalpy, BTU/lbm
k	Thermal conductivity of salt, BTU/ft-hr-^{o}F
k_1, k_2 k_3, k_4	Thermal conductivities on the element surfaces; $r=r_i+\Delta r/2$, $\psi=\psi_i+\Delta\psi/2$, $r=r_i-\Delta r/2$, $\psi=\psi_i-\Delta\psi/2$
k_f	Thermal conductivity of heat pipe wall, BTU/ft-hr-^{o}F
L	Radius of liquid solid interface, ft
L_f	Radius of liquid solid interface when solidification is complete, ft
ℓ	Dimensionless frontal radius
ℓ_f	Dimensionless radius when solidification is complete
$\hat{h}$	Unit vector normal to the surface and directed outward from the control volume
P	Thickness of heat pipe wall, ft
q''	Heat flux rate, BTU/ft^2-hr
R	Radial distance, ft
R_I	Radius of outer surface of heat pipe, ft
R_O	Inner radius of outter cannister wall, ft
r	Dimensionless radius
S_{T_e}	Stefan Number
T	Temperature, ^{o}F
T_F	Fusion temperature of salt, ^{o}F
ΔT	Temperature drop across heat pipe wall, ^{o}F
t	Time, hr
v	Dimensionless volume = volume/πR_I^2

α	Thermal diffusivity of salt, ft^2/hr
θ	Dimensionless temperature
δ	Density of salt, lbm/ft^3
τ	Dimensionless time
τ_f	Dimensionless time when solidification is complete
ψ	Angle from Fin centerline

LIST OF TABLES

SECTION I
INTRODUCTION

In many applications, thermal energy is available in an intermittant form, and demand for this energy does not necessarily coincide with availability. Solar energy, for example, is not available at night, and similarly wind powered electrical systems function only when the wind velocity is adequate. Also, the greater cost of commercial electrical power during periods of peak demand is another application in which energy storage could be used to good advantage. For these reasons, systems which can economically store large quantities of thermal energy with suitable recovery on demand are of increasing interest.

There are numerous energy storage systems available, at present, such as batteries, hydrogen storage/heat engine, flywheels, etc. Systems utilizing the phase change of materials having a high latent heat of fusion are also being evaluated at present and are of particular interest because:

1) heat transfer occurs at nearly constant temperatures and

2) large quantities of thermal energy may be stored in a relatively small volume.

The basic problem with systems utilizing the phase change of materials is analytically determining the melting and solidifying characteristics of a given system. The problem is inherently nonlinear and other than a few exact solutions the problem must be solved using numerical or analogue techniques. Carslaw and Jaeger[1] discuss the few exact solutions available as well as some approximate solutions. Kreith and Romie[2] present solutions for the cases of uni-dimensional cylinders, spheres and semi-infinite solids for the condition that the temperature gradient in the solid on solidification is constant or that the velocity of the solid/liquid interface remains constant. Kreith and

Romie[2] also discuss analogue computer solutions for semi-infinite solids. The bulk of the problems involving phase change have been solved by numerical techniques and include a variety of applications and problems such as continuous casting of cylindrical ingots[3], two-dimensional freezing including convection effects in the liquid region[4], and axisymmetric semi-infinite hollow-cylinder with convective heat transfer at the inner radius[5].

SECTION II
OBJECTIVE AND SCOPE

The purpose of this investigation is to determine the thermal characteristics of energy-storage systems utilizing a cylindrical annulus to contain a phase change material. The outer surface is insulated and the inner surface (a heat pipe) is subjected to a constant heat flux. Additionally the influence of longitudinal fins within the annulus, with regard to heat transfer characteristics, is to be assessed. The fins will be symmetrically positioned in the circumferential direction.

Since a solution to a problem in solidification is also a solution to the corresponding problem in melting only solidification will be considered. The system is of sufficient length, in conjunction with heat pipe characteristics, that axial gradients will be neglected. The phase change material will be assumed to have a distinct fusion temperature and initially the liquid is uniformly at the fusion temperature. Additionally the thermal properties will be assumed to be uniform within each phase and mean values will be used for each phase. Convection within the fluid as well as radiation within both phases will be neglected.

SECTION III
SYSTEM DESCRIPTION

The thermal storage system consists of a cannister surrounding a heat pipe. The annulus formed by the cannister and the heat pipe contains 4.1705 lb. of the thermal storage material which is LiF - MgF_2 - KF. The cannister and heat pipe walls are made of .065" thick 304 Stainless Steel. The outside diameters are 1.5" for the heat pipe and 3.0" for the cannister. The geometry of this configuration is illustrated in Figure 1. Thermal properties assumed for the materials are presented in Table I. In addition there are two finned configurations containing three and six fins of 1/16" thick 304 Stainless Steel. The fins are parallel to, and project radially outward from, the longitudinal axis of the heat pipe. The configurations with three and six fins contain 3.9985 lb. and 3.8742 lb. of the thermal storage material, respectively. The finned configurations are illustrated in Figure 2.

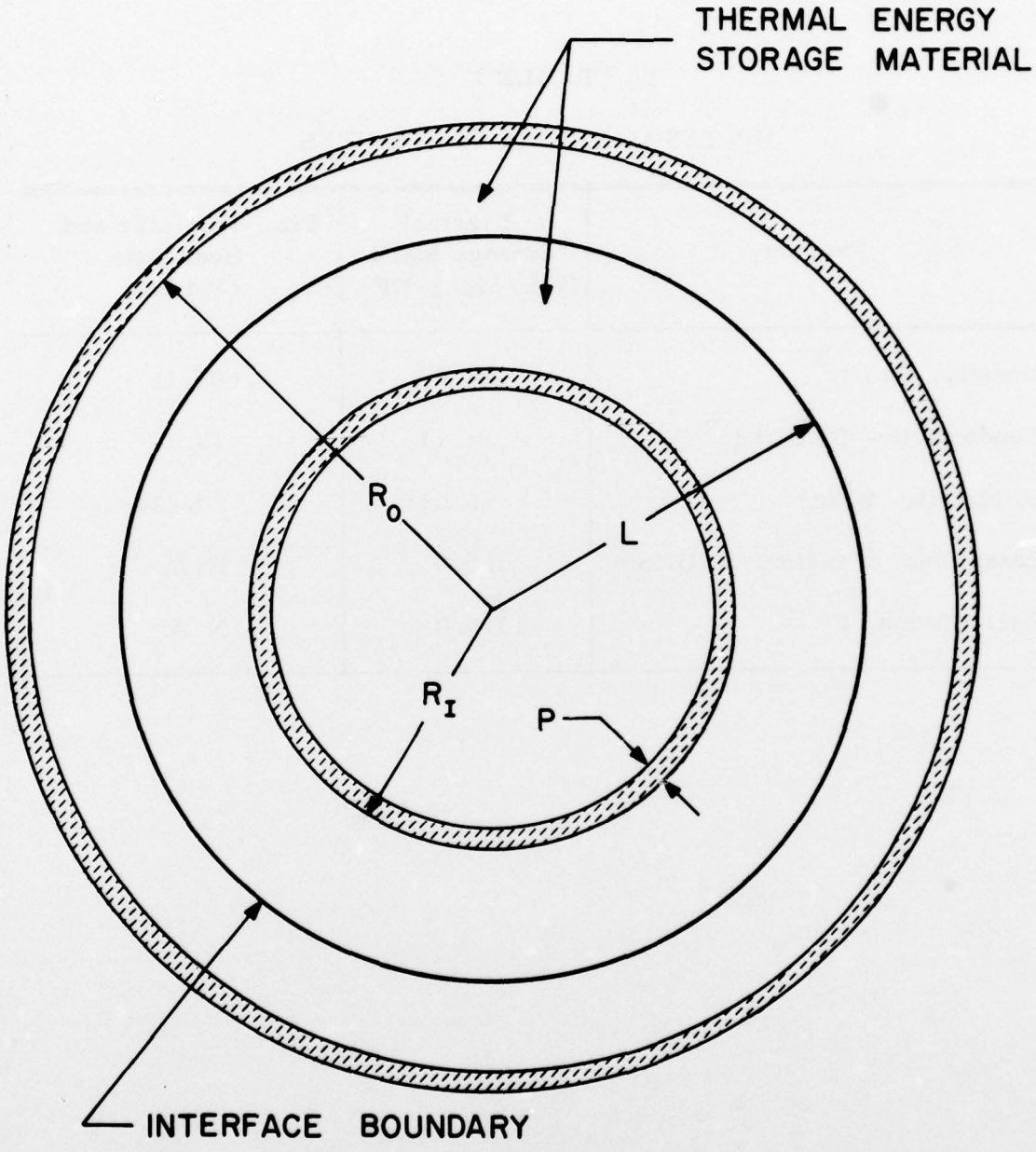

Figure 1. Axisymmetric HP/TES System Geometry

TABLE 1

HP/TES MATERIAL PROPERTIES

Property	Thermal Storage Mat'l ($LiF-MgF_2-KF$)	Fin, Cannister and Heat Pipe (304 SS)
Density, lbm/ft^3	181.0	501.12
Conductivity, BTU/ft lb° F	4.11	13.5
Diffusivity, ft^2/hr	0.035	0.224
Latent heat of fusion, BTU/lbm	350.0	N.A.
Fusion temp, F	1310.0	N.A.

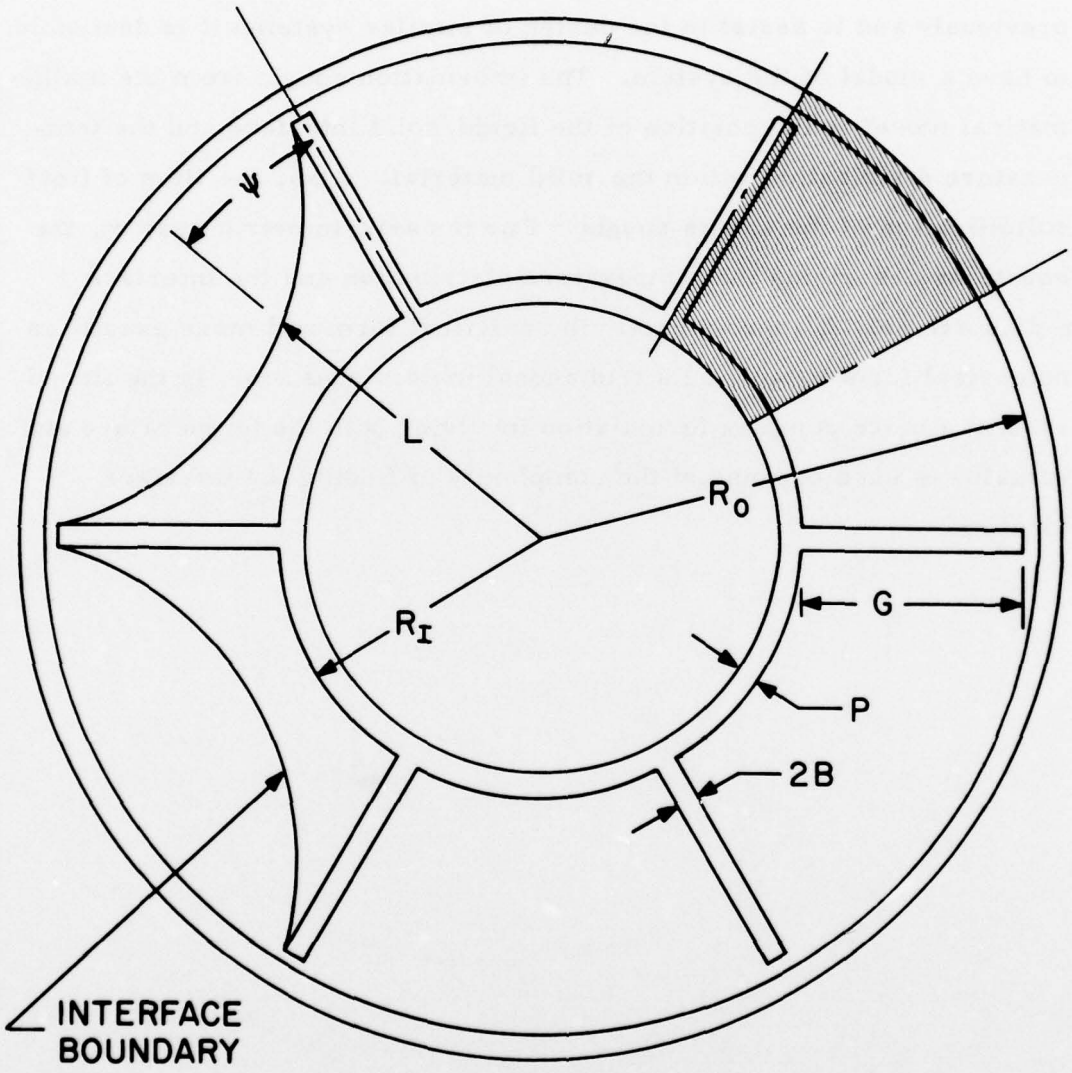

Figure 2. Finned HP/TES System Geometry

SECTION IV
ANALYTICAL APPROACH

In order to evaluate experimental data for the system described previously and to assist in the design of similar systems it is desirable to have a model of the system. The information sought from the mathematical model is the position of the liquid/solid interface and the temperature distribution within the solid material. Also, the time of final solidification of the salt is sought. For the axisymmetric system, the equations describing the temperature distribution and the interface radius are solved approximately in analytical form and more exactly in numerical form by solving a tridiagonal matrix equation. In the finned system a more complex formulation involving both the temperature and enthalpy is used because of the complexity of finding the interface location.

SECTION V

ANALYSIS OF AXISYMMETRIC SYSTEM

Formulation of Equations

In the axisymmetric model it is assumed that variations along the longitudinal axis of the system are negligible. Heat losses from the outer shell of the cannister are assumed negligible as is heat transfer by radiation between the hot outer portion of the system and the cold solidified salt. The entire system is assumed to be initially at the fusion temperature of the salt. The heat flux rate at the inner surface is assumed to be constant with respect to time and over the surface of the heat pipe. The change in internal energy of the heat pipe wall as heat is extracted from the system is ignored.

Under these assumptions the temperature distribution within the salt is governed by the unidimensional transient heat conduction equation:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial R^2} + \frac{1}{R} \frac{\partial T}{\partial R} \qquad (1)$$

where T = temperature, F

t = time, hr

R = radial distance, ft

α = thermal diffusivity of salt, ft^2/hr

Equating the heat flux rate at the inner surface with the rate at which heat is conducted into a unit area of the surface yields:

$$k \frac{\partial T}{\partial R} = q'' \qquad \text{for } R = R_I,\ t > 0 \qquad (2)$$

where k = thermal conductivity of the salt, BTU/ft - hr - F

$\dot{q}''$ = heat flux rate, BTU/ft^2 - hr

R_I = radius of outer surface of heat pipe, ft

By assumption the initial temperature distribution is described by

$$T(R,t) = T_F \qquad \text{for } R > R_I,\ t = 0 \tag{3}$$

where T_F = fusion temperature of salt, F.

Also, at the interface, the temperature is always at the fusion temperature:

$$T(R,t) = T_F \qquad \text{for } R = L,\ t > 0 \tag{4}$$

where L = radius of liquid solid interface, ft.

Equations (1) through (4) constitute a partial differential equation of second order in space and first order in time along with the necessary initial condition and boundary conditions. However, since L is an unknown function of time, the problem is incomplete.

To determine L, two approaches are considered. In the first, the heat created by fusion at the interface is equated with the heat conducted from the interface to yield:

$$2\pi L \rho h_{s\ell} \frac{\partial L}{\partial t} = 2\pi L\ k\ \frac{\partial T}{\partial R} \qquad \text{for } R = L,\ t > 0 \tag{5}$$

where ρ = density of salt, lbm/ft^3

$h_{s\ell}$ = latent heat of fusion of salt, BTU/lbm

Equation (5) along with the initial condition:

$$L(t) = R_I \text{ for } t = 0 \tag{6}$$

is sufficient to complete the formulation of the problem partially posed by Equations (1) through (4).

A second approach to determine L may be taken by equating the change in internal energy of the salt to the heat conducted out of the salt at the inner surface. This may be expressed as

$$\rho\pi(L^2 - R_I^2)h_{s\ell} - \int_{R_I}^{R=L} \rho c(T - T_F)2\pi R\,dR = \dot{q}''t(2\pi R_I) \tag{7}$$

where c is the specific heat of the salt, BTU/lbm^{o}F.

In Equation (7) the first term is the internal energy from fusion of the salt, the second term is the change in internal energy from the solid salt at the fusion temperature and the term on the right hand side is the heat extracted from the inner surface.

There will be a temperature drop across the heat pipe wall since heat is conducted across it. Since the change in internal energy of the wall is neglected this may be found from the steady state heat conduction equation to be:

$$\Delta T = \frac{\dot{q}'' R_I}{k_f} \text{Ln} \left\{ \frac{R_I}{R_I - p} \right\} \tag{8}$$

where ΔT = temperature drop across heat pipe wall, F

k_f = thermal conductivity of heat pipe wall, BTU/ft - hr - F

p = thickness of heat pipe wall, ft.

Systems such as that described are seldom completely filled with solid salt in order to avoid excessive hydrostatic pressure since the salts are more dense in solid form than liquid. The ratio of the volume of solidified salt to the cannister volume may be written:

$$f = \frac{L_f^2 - R_I^2}{R_o^2 - R_I^2} \tag{9}$$

where f = volume ratio of solid salt to cannister

R_o = inner radius of outer cannister wall, ft

L_f = radius of liquid-solid interface when solidification is complete.

To generalize the solutions, it is desirable to express the preceeding equations in terms of dimensionless variables. Substituting the variables

$$\theta = k(T-T_F)/\dot{q}''R_I;\ r = R/R_I;\ \ell = L/R_I;\ \tau = \alpha t/R_I^2 \tag{10}$$

into Equations (1) thru (7) yields, respectively

$$\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} = \frac{\partial\theta}{\partial\tau} \tag{11}$$

$$\frac{\partial\theta}{\partial r} = 1. \qquad \text{for } r = 1,\ \tau > 0 \tag{12}$$

$$\theta(r,\tau) = 0. \qquad \text{for } r > 1,\ \tau = 0 \tag{13}$$

$$\theta(r,\tau) = 0. \qquad \text{for } r = \ell,\ \tau > 0 \tag{14}$$

$$\frac{\partial\ell}{\partial\tau} = S_{T_e}\frac{\partial\theta}{\partial r} \qquad \text{for } r = \ell,\ \tau > 0 \tag{15}$$

$$\ell(\tau) = 1. \qquad \tau = 0 \tag{16}$$

$$\frac{\ell^2 - 1}{2\,S_{T_e}} - \int_1^{\ell} \theta r dr = \tau \tag{17}$$

Equation (16) may be solved for ℓ to yield

$$\ell = \sqrt{1 + 2\,S_{T_e}\cdot(\tau + \int_1^{\ell} \theta r dr)} \tag{18}$$

In Equations (15), (17), and (18):

$$S_{T_e} = \dot{q}''R_I/\alpha\rho h_{s\ell} \tag{19}$$

where S_{T_e} = Stefan Number

τ = dimensionless time

ℓ = dimensionless frontal radius

r = dimensionless radius

θ = dimensionless temperature

It may be seen from Equations (11) through (16) that the solution for θ and ℓ in terms of r and τ is characterized by the single dimensionless parameter S_{T_e}. This is also true if Equation (18) is used rather than Equations (15) and (16) in conjunction with (11) through (14) to formulate the problem.

To find the time when solidification is complete, from Equation (9)

$$\ell_f = \sqrt{f \frac{(R_o^2 - R_I^2)}{R_I^2} + \frac{R_I^2}{R_I^2}} = \sqrt{1 + f(r_o^2 - r_I^2)} \tag{20}$$

and from Equation (17)

$$\tau_f = \frac{\ell_f^2 - 1}{2 S_{T_e}} - \int_1^{\ell_f} \theta r dr \tag{21}$$

where ℓ_f = dimensionless radius when solidification is complete

τ_f = dimensionless time when solidification is complete.

Solution

Two different approaches to solving the system of equations representing the axisymmetric system will be considered. In the first, the change in internal energy of the solidified salt is neglected and an approximate analytical solution is found by solving Equations (11) through (15) subject to this assumption. To obtain the second solution, Equations (11) through (14) are expressed in finite difference form. The resulting tridiagonal matrix system is solved in conjunction with Equation (18) to find the temperature distribution and interface motion.

Approximate Analytical Solution

If the change in internal energy of the solid salt is neglected in comparison with the heat of fusion Equation (11) may be expressed:

$$\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} = 0 \tag{22}$$

Then the expression $\theta = C_1 \ln r + C_2$ is seen to satisfy Equation (22). Applying the boundary conditions (12) and (14) gives the solution:

$$\theta = \ln\left(\frac{r}{\ell}\right) \tag{23}$$

Substituting (23) into (15) yields

$$\frac{\partial \ell}{\partial \tau} = \frac{S_{T_e}}{\ell} \tag{24}$$

In Equation (24) we may separate variables and integrate to find an expression for ℓ:

$$\ell = \sqrt{1 + 2\, S_{T_e}\, \tau} \tag{25}$$

Note that the expression (25) satisfies Equation (18) if:

$$\int_1^{\ell} \theta r dr = 0 \tag{26}$$

Equation (26) is merely a restatement of the assumption that the change in internal energy of the salt is negligible which will later be shown to be quite accurate for the system being considered.

Numerical Solution

The analytical solution derived above for the axisymmetric system is an accurate solution where the heat of fusion is numerically much larger than the specific heat of the storage material. If, however, the latent heat is not significantly larger numerically than the specific heat, then the assumption made to obtain that solution may be too strong to yield realistic results. For this reason a numerical solution in which the internal energy of the salt was accounted for was derived and is presented here.

Let the temperature distribution at time t^k be known in the form of

$$\theta i^k = \theta(r_i, t^k) \quad ; \quad 1 \leq i \leq n \tag{27}$$

Then if $\ell_{k+1} = \ell(t^{k+1})$ is known we may find θi^{k+1} $1 \leq i \leq n+1$ from the tridiagonal matrix equation derived in Appendix A which is of the form

$$\begin{bmatrix} a_1 & b_1 & & & \\ c_2 & a_2 & b_2 & & \\ & c_3 & a_3 & b_3 & \\ & & & & \\ & & & c_n & a_n \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \\ \\ \theta_n \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \\ \\ d_n \end{bmatrix} \tag{28}$$

The procedure to calculate the temperature distribution is illustrated by the flow chart in Figure 3. First, a time increment $\Delta\tau$ is selected and r(2) is calculated from Equation (25) as:

$$r(2) = \ell_1 = \ell(\Delta\tau) = \sqrt{1+2S_{T_e}\Delta\tau} \tag{29}$$

and

$$\theta_1^1 = \theta(r=1) = \theta(r(1)) = \ell n(\frac{1}{r(2)}) \tag{30}$$

and

$$\theta_2^1 = 0 \tag{31}$$

where the θ's are found from Equation (23).

From this point on, the procedure to find $\theta i^{k+1} = \theta i^k$ is the same for each time step. Initially, it is assumed that $\theta i^{k+1} = \theta i^k$ and ℓ_{k+1} is calculated from Equation (18) where

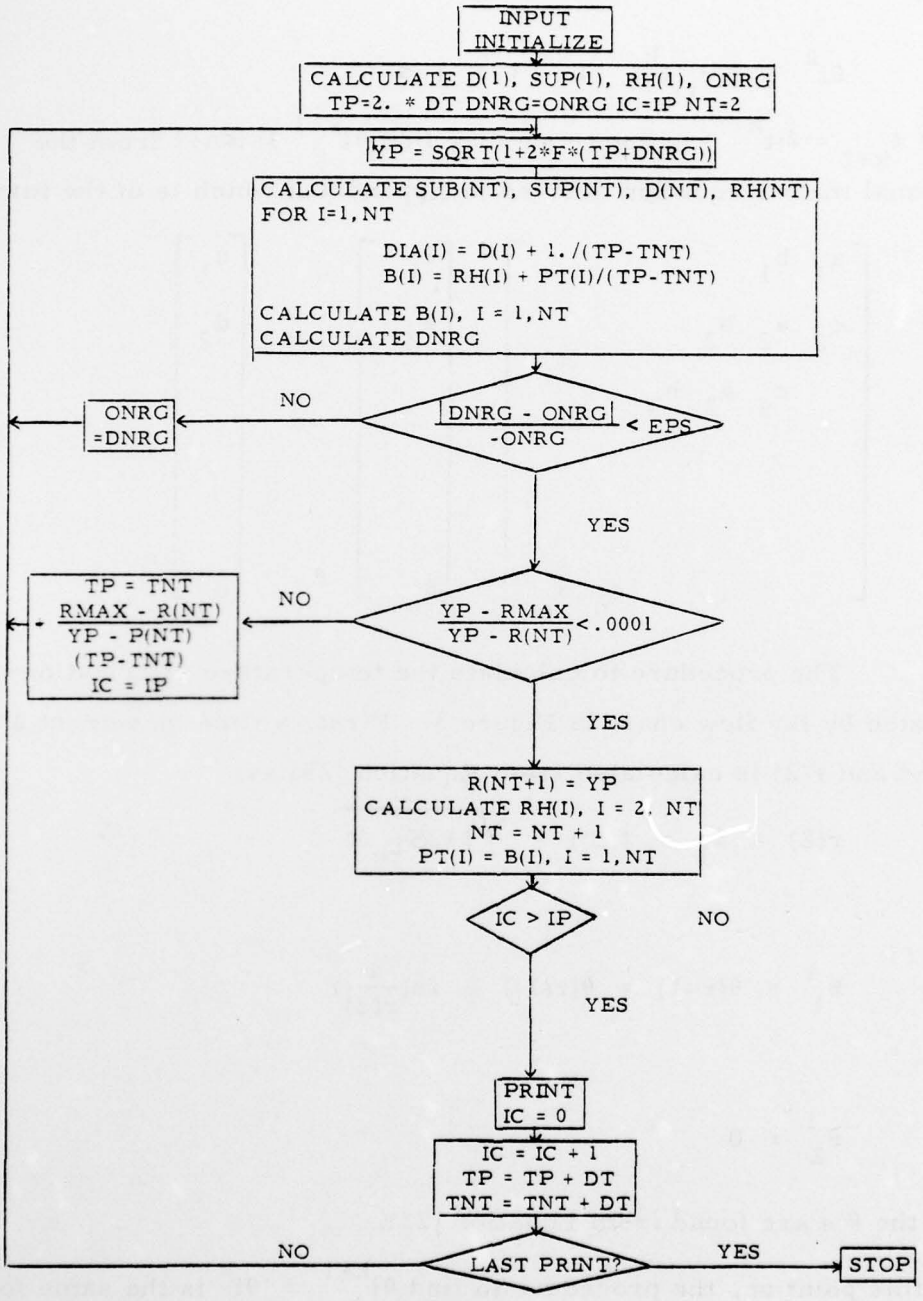

Figure 3. Flow Chart of Temperature Model of Axisymmetric HP/TES System

$$\int_1^{\ell} \theta r dr = \sum_{i=2}^{n} \{(\theta_i^{k+1} r_i + \theta_{i-1}^{k+1} r_{i-1})/2.\}(r_i - r_{i-1}) + \theta_n^{k+1} r_n (r_{n+1} - r_n)/2. \quad (32)$$

then θ_i^{k+1}, i=1, n are calculated from Equations (28). This temperature distribution is used to calculate a new ℓ which in turn is used to calculate a new temperature distribution. This iterative procedure is continued until the change in ℓ meets an acceptable error criteria at which point the temperature distribution for the mew time has been calculated, and the procedure is initiated for the next time. The time increment is held uniform except that for the final time step, $\Delta\tau$ is shortened so that $\ell(\tau_f) = \ell_f$ from Equation (20). The computer program developed to calculate the axisymmetric temperature distribution is documented in Appendix B.

SECTION VI
ANALYSIS OF FINNED SYSTEM

Formulation of Equations

An approach similar to that used for the axisymmetric case could be used to model the finned system. However, the boundary would be much more difficult to locate since its location varies with the angle from the fin centerline. Instead an approach involving both the enthalpy and the temperature of the salt employed by Shamsunder and Sparrow[6] is used. In this approach, the solid liquid boundary is found as one of the results of the solution.

Since there was some uncertainty as to the accuracy of this method the method was first applied to the axisymmetric case and the results were used to examine the accuracy of the method. This formulation will be presented as an aid to understanding the approach. Then the method will be used to formulate the finned problem.

In this approach the region of interest is again divided into elements of finite size. If there are no sources of energy inside the control volume, pressure does not vary with time, and no external work is done on the control volume the net rate of increase of internal energy must equal the rate at which heat is conducted into the control volume. Then enthalpy and temperature are related in the law of conservation of energy for the element.

$$\frac{d}{dt} \iiint_V \rho h dv = \int_A k \text{ grad } T \cdot \hat{n} dA \qquad (33)$$

where h = specific enthalpy BTU/lbm

$\hat{n}$ = unit vector normal to the surface and directed outward from the control volume.

This may also be written in terms of dimensionless variables

$$\eta = \frac{1}{\rho_s \Delta v} \iiint_v \frac{\rho(h-h_T^*)}{hs\ell}\, dv \equiv \text{dimensionless enthalpy} \tag{34}$$

$$\theta = \frac{k_f(T-T^*)}{\alpha_f \rho_s hs\ell} \equiv \text{dimensionless temperature} \tag{35}$$

and enthalpy may be related to temperature by

$$h-h_s^* = \begin{cases} c(T-T_F) & , \quad T < T_F \\ c(T-T_F)+hs\ell & , \quad T > T_F \end{cases} \tag{36}$$

Axisymmetric Formulation and Solution

Let the region $R_I < R < R_0$ be divided into n elements, each a concentric ring of width ΔR. Let the thickness of the cannister wall be $n_f \Delta R$. Then, initially $T = T_F$ and the thermal storage material is all liquid so that initially

$$\begin{aligned} \theta &= 0. \qquad i = 1, m \\ \eta &= 0. \qquad i \le n_f \\ \eta &= 1. \qquad n_f < i \le m \end{aligned} \tag{37}$$

where $m = n + n_f$ total number of nodes.

Writing (33) in terms of the elements gives

$$\begin{aligned} &\rho_a \cdot 2\pi R \Delta R h_{sf} \frac{d}{dt} \frac{1}{\rho_a 2\pi R \Delta R} \int_{R-\frac{\Delta R}{2}}^{R+\frac{\Delta R}{2}} \frac{\rho(h-h_s^*)}{h_{sf}}\, dR \\ &- k_1 \frac{\partial T}{\partial r} 2\pi(R+\tfrac{\Delta R}{2}) - k_2 \frac{\partial T}{\partial r} 2\pi(R-\tfrac{\Delta R}{2}) \\ &- \left(k_1 \frac{\partial(k_a(T-T^*)/\alpha\rho h_{sf})}{\partial R} 2\pi(R+\tfrac{\Delta R}{2}) - k_2 \frac{\partial(k_a(T-T^*)/\alpha\rho h_{sf})}{\partial R} 2\pi(R-\tfrac{\Delta R}{2})\right) \frac{\alpha_a \rho_a h_{sf}}{k_a} \end{aligned} \tag{38}$$

where k_1 = thermal conductivity at outer edge of element

k_2 = thermal conductivity at inner edge of element.

Expressing (38) in terms of the dimensionless variables r, τ, θ, and η yields

$$\Delta r \frac{d\eta}{d\tau} = \left(1+\frac{\Delta r}{2r_i}\right) \frac{k_1}{k_s} \frac{\partial\theta}{\partial r}\bigg|_{r=r_i+\frac{\Delta r}{2}} - \left(1-\frac{\Delta r}{2r_i}\right) \frac{k_2}{k_s} \frac{\partial\theta}{\partial r}\bigg|_{r=r_i-\frac{\Delta r}{2}}$$

which can be expressed in terms of finite differences

$$\frac{(\Delta r)^2}{\Delta\tau} \left(\eta_i^k - \eta_i^{k-1}\right) = \left(1+\frac{\Delta r}{2r_i}\right)\frac{k_1}{k_s} \left(\theta_{i+1}^k - \theta_i^k\right) - \left(1-\frac{\Delta r}{2r_i}\right) \frac{k_2}{k_s} \left(\theta_i^k - \theta_{i-1}^k\right) \tag{39}$$

If the element is considered to be at a uniform temperature, substitute Equation (36) into (34) to find the relation between temperature and enthalpy:

$$\eta = \frac{1}{\rho_s \Delta v} \int_{r_i-\frac{\Delta r}{2}}^{r_i+\frac{\Delta r}{2}} \frac{\rho c(T-T^*)}{h_{s\ell}}\, dr$$

$$= \frac{\rho c}{\Delta v} \frac{\alpha_s}{k_s} \frac{k_s(T-T^*)}{\alpha_s \rho_s h_{s\ell}} \Delta v = \frac{\alpha_s k}{\alpha k_s}\, \theta$$

$$\theta = \frac{\alpha k_s}{\alpha_s k}\, \eta \qquad \eta < 0$$

$$= 0 \qquad \eta > 0 \tag{40}$$

Now, rearranging Equation (39) yields

$$\frac{(\Delta r)^2}{\Delta\tau} \eta_i^k + \left\{\left(1+\frac{\Delta r}{2r_i}\right)\frac{k_1}{k_s} + \left(1-\frac{\Delta r}{2r_i}\right)\frac{k_2}{k_s}\right\} \theta_i^k$$

$$= \frac{(\Delta r)^2}{\Delta\tau} \eta_i^{k-1} + \left(1+\frac{\Delta r}{2r_i}\right)\frac{k_1}{k_s}\, \theta_{i+1}^k + \left(1-\frac{\Delta r}{2r_i}\right)\frac{k_2}{k_s}\, \theta_{i-1}^k \quad ; \quad i = 2, \ldots, m \tag{41}$$

$$\text{where } k_1 = \begin{cases} k_f & h < h_f \\ 2\,k_f k_s/(k_f+k_s) & h = h_f \\ k_s & h > h_f \end{cases}$$

$$k_2 = \begin{cases} k_f & h \le h_f \\ 2\,k_f k_s/(k_f+k_s) & h = h_f + 1 \\ k_s & h > h_f + 1 \end{cases}$$

Substitute for θ from Equation (40) into the left hand side of Equation (41) to obtain

$$\left[\frac{(\Delta r)^2}{\Delta\tau} + \frac{\alpha k_s}{\alpha_s k}\left\{(1+\frac{\Delta r}{2r_i})\frac{k_1}{k_s} + (1-\frac{\Delta r}{2r_i})\frac{k_2}{k_s}\right\}\right]\eta_i^k$$

$$= \frac{(\Delta r)^2}{\Delta\tau}\eta_i^{k-1} + (1+\frac{\Delta r}{2r_i})\frac{k_1}{k_s}\theta_{i+1}^k + (1-\frac{\Delta r}{2r_i})\frac{k_2}{k_s}\theta_{i-1}^k \quad , \eta_i^k < 0 \tag{42}$$

and

$$\frac{(\Delta r)^2}{\Delta\tau}\eta_i^k = \frac{(\Delta r)^2}{\Delta\tau}\eta_i^{k-1} + (1+\frac{\Delta r}{2r_i})\frac{k_1}{k_s}\theta_{i+1}^k + (1-\frac{\Delta r}{2r_i})\frac{k_2}{k_s}\theta_{i-1}^k \quad , \eta_i^k > 0$$

At the inner surface of the innermost node, the heat flux is known so that Equation (33) may be written

$$\frac{\rho_s 2\pi R\Delta R h_{s_\ell}\alpha_s}{R_I^2}\frac{d\eta}{d\tau} = \frac{k_1}{k_s}\frac{\partial\theta}{\partial R}2\pi(R+\frac{\Delta R}{2})\alpha_s\rho_s h_{s_\ell} - 2\pi\dot{q}''R_I$$

where $\dot{q}'' = \dfrac{\dot{q}'}{2\pi R_I}$

$$\frac{(\Delta r)^2}{\Delta\tau}(\eta_1^k - \eta_1^{k-1}) = \frac{k_1}{k_s}(1+\frac{\Delta r}{2r_1})(\theta_2^k - \theta_1^k) - \frac{\dot{q}''R_I}{\alpha_s\rho_s h_{s_\ell}}\Delta r \tag{43}$$

Notice that the parameter S_{T_e} has again appeared on the right hand side of Equation (43). Since, in the previous analysis we concluded that this was the driving parameter for the solution, it is logical it should also appear in this formulation. Equation (43) may be rearranged similar to Equation (42) to yield

$$\left\{\frac{(\Delta r)^2}{\Delta\tau} + \frac{\alpha k_s}{\alpha_s k}\left(1+\frac{\Delta r}{2r_i}\right)\frac{k_l}{k_s}\right\}\eta_l^k = \frac{(\Delta r)^2}{\Delta\tau}\eta_l^{k-1} + \frac{k_l}{k_s}\left(1+\frac{\Delta r}{2r_i}\right)\frac{\alpha k_s}{k\alpha_s}\theta_{i+1}^k - S_{T_e}\Delta r \tag{44}$$

$$\text{If } \eta_l^k < 0$$

Equations (42) and (44) are solved by Gauss-Seidel iteration with over-relaxation. The procedure is to first calculate the right hand side. Since the coefficient of η_i^k is always positive the sign of the right hand side is the same as that of η so that the appropriate coefficient may be determined. η is then calculated by dividing the right hand side by the coefficient of η. When the temperature distribution has been found with sufficient accuracy the volume solidified and the change in internal energy are calculated from:

$$V_s = \sum_{i=1}^{m} 2\pi r_i(\eta_i^o - \max(\eta_i^k, 0.)) \tag{45}$$

$$E_t = \sum_{i=1}^{m} 2\pi r_i(\eta_i^o - \eta_i^k) \tag{46}$$

Finned System Formulation and Solution

The model for the finned system is set up very similarly to that for the axisymmetric system. Since the temperature may vary with angle as well as radius two spatial dimensions must be included in the model. Because of the symmetry assumed, only the shaded region in Figure 2 need be modeled and the heat flux across the boundaries $\psi = 0$ and $\psi = \pi/(\text{no. of fins})$ is known to be zero. Also because heat losses are neglected there is no heat flux across the outer surface. The heat flux across the inner boundary is specified as part of the problem so that the heat flux is known over the entire boundary of the region.

We write the enthalpy equation for an element containing a volume $f\Delta v$ of fin material and a volume $(1-f)\Delta v$ of salt and having thermal conductivities k_1, k_2, k_3, and k_4 on the surfaces $r = r_i + \frac{\Delta r}{2}$, $\psi = \psi_i + \frac{\Delta\psi}{2}$,

$r = r_i - \frac{\Delta r}{2}$, and $\psi = \psi_i - \frac{\Delta\psi}{2}$ respectively. Then Equation (33) may be written

$$\frac{(\Delta r)^2}{\Delta\tau}\eta^k_{i,j} + \left[(1+\frac{\Delta r}{2r})\frac{k_1}{k_n} + \frac{k_2}{k_n}\left(\frac{\Delta r}{r\Delta\psi}\right)^2 + (1-\frac{\Delta r}{2r})\frac{k_3}{k_n} + \frac{k_4}{k_n}\left(\frac{\Delta r}{r\Delta\psi}\right)^2\right]\theta^k_{i,j}$$

$$- \frac{(\Delta r)^2}{\Delta\tau}\eta^{k-1}_{i,j} + (1+\frac{\Delta r}{2r})\frac{k_1}{k_n}\theta^k_{i+1,j} + \frac{k_2}{k_n}\left(\frac{\Delta r}{r\Delta\psi}\right)^2\theta^k_{i,j+1}$$

$$+ (1-\frac{\Delta r}{2r})\frac{k_3}{k_n}\theta^k_{i-1,j} + \frac{k_4}{k_n}\left(\frac{\Delta r}{r\Delta\psi}\right)^2\theta^k_{i,j-1} \qquad (47)$$

Now we use Equation (34) to establish the relation between η and θ assuming the element at uniform temperature

$$\eta = \frac{1}{\rho_s \Delta v}\iiint \frac{\rho(h-h_s)}{h_{s_\ell}}\,dv$$

$$= \frac{\rho_f c_f (T-T_s^*)}{\rho_s \Delta v h_{s_\ell}} f\Delta v + \frac{\rho_s c_s}{\rho_s \Delta v}\frac{T-T_s^*}{h_{s_\ell}}(1-f)\Delta v$$

$$= \frac{k_s (T-T_s^*)}{\alpha_s \rho_s h_{s_\ell}}\left\{f\frac{\alpha_s}{k_s}\rho_f c_f + (1-f)\right\}$$

$$\theta = \begin{cases} \dfrac{\eta}{1 - f(1-\dfrac{\alpha_s k_f}{k_s \alpha_f})}, & \eta < 0 \\ 0., & 0 < \eta < 1. \end{cases} \qquad (48)$$

Equation (47) must be modified for boundary elements similar to the axisymmetric case. The iterative procedure for solution is illustrated by the flow chart in Figure 4. Here, Equation (47) is expressed:

$$\eta^k_{i,j} = \{Di\eta^{k-1}_{i,y} + C1(I,J)\cdot\eta^k_{i+1,j} + C2(I,J)\cdot\eta^k_{i,j+1} + C3(I,J)\cdot\eta^k_{i-1,j} + C4(I,J)\cdot\eta^k_{i,j-1}/C5(I,J) \qquad (49)$$

$$C5(I,J) = D + (C1(I,J)+C2(I,J)+C3(I,J)+C4(I,J)\}/\{1-f\left(1-\frac{\alpha_s k_f}{\alpha_f k_s}\right)\}$$

when $\eta^k_{i,j} = 0.$

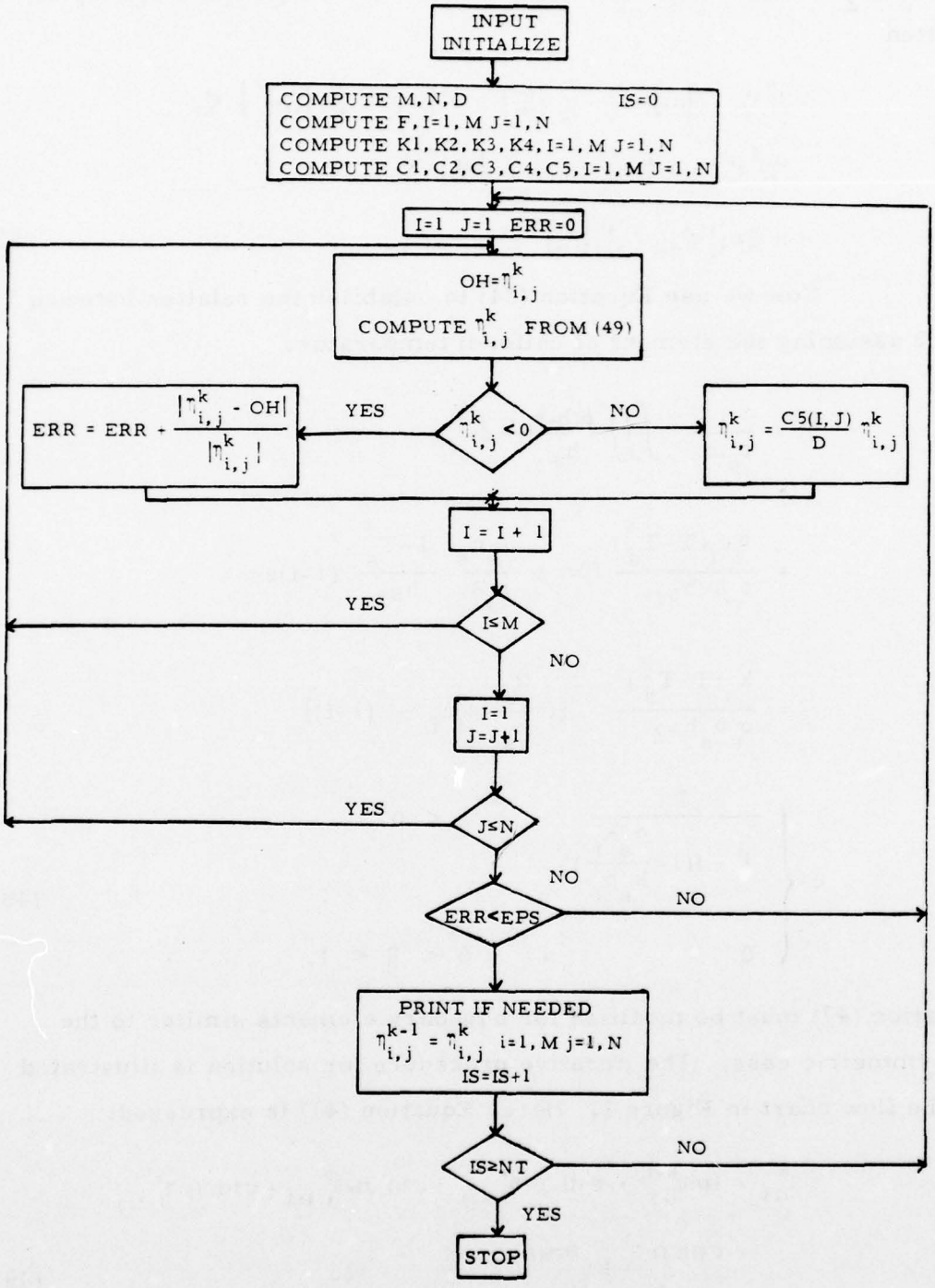

Figure 4. Flow Chart for Enthalpy Model of Finned HP/TES System

SECTION VII
RESULTS AND DISCUSSION

Results with Axisymmetric Case

The approximate analytical solution as previously derived in dimensionless form is

$$\theta = \ell n\,(r/\ell) \tag{23}$$

$$\ell = \sqrt{1 + 2\,S T_e\,\tau} \tag{25}$$

If this solution is correct, the change in energy from the heat fusion is

$$\pi R_I^2(\ell^2 - 1)\rho h_{s_\ell} = \Delta E.$$

and the change in internal energy of the salt, which was neglected is

$$\Delta E_2 = \int_{R_I}^{R_0} \frac{k}{\alpha}(T - T_F)\,2\pi R dR$$

$$= \int_1^{\ell} \frac{2\pi \dot{q}'' R_I^3}{\alpha}\,\frac{k(T - T_F)}{q'' R_I}\,rdr$$

$$= \frac{2\pi q'' R_I^3}{\alpha} \int_1^{\ell} \theta r dr \tag{50}$$

Substituting Equations (23) into (50) and integrating yields

$$\Delta E_2 = \frac{2\pi \dot{q}'' R_I^3 \ell^2}{\alpha}\left\{\frac{1 - \ell^2}{4} + \frac{1}{2}\,Ln(\ell)\right\} \tag{51}$$

The ratio of the neglected energy change ΔE_2 to the change from the heat fusion ΔE_1 is

$$\frac{\Delta E_2}{\Delta E_1} = \frac{q'' R_I}{\alpha \rho h_s \ell} \frac{\ell^2}{\ell^2 - 1} \mathrm{Ln}(\ell) - \frac{1}{2}$$

$$= S T_e \frac{\ell^2}{\ell^2 - 1} \mathrm{Ln}(\ell) - \frac{1}{2} \qquad (52)$$

In the worst case (heat rate = 1.5 kw, $\ell = \ell_f$) the neglected heat is only 10.8% of the heat included so that the approximate solution is fairly accurate for the system. Since the change in internal energy of the solid salt is such a small part of the heat extracted from the system, the solidification times predicted by this simple model will be very accurate, even if part of the solidification takes place on the outer wall due to heat losses. In this case however, the temperature drop would be less than that predicted since the heat of fusion would not have to be conducted across as thick a layer of solidified salt.

Before meaningful results could be obtained for the numerical solutions for the axisymmetric case, it was necessary to determine convergence. In this program the increments in both radius and time are determined by the parameter Δt and was for each case half that of the preceding case. Once the temperatures agreed for consecutive time increments to three significant digits, convergence was assumed to be adequate. For the runs with other heat rates, Δt was adjusted to maintain the same increment in radius.

In Figure 5 temperature is plotted as a function of radius for several different times for the axisymmetric system with a heat extraction rate of 1 kw. Both the analytical and the numerical solutions are shown. It may be seen that at any given time the numerical solution predicts a slightly smaller interface radius than does the analytical

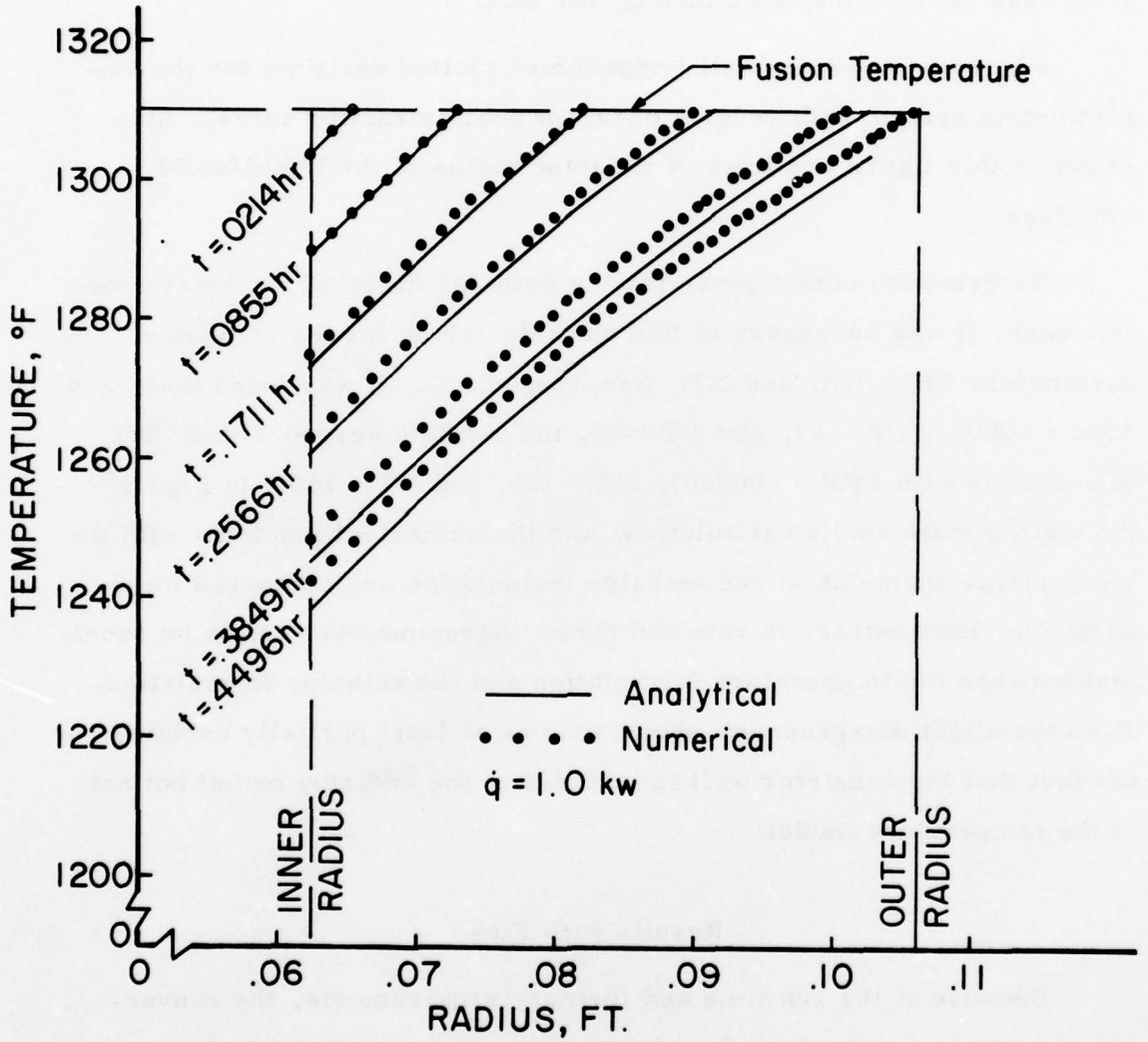

Figure 5. Temperature Distribution As A Function of Radial Position in HP/TES System (Axisymmetrical Case)

solution. This is because a small amount of the heat extracted comes from the change in internal energy of the solid salt. Therefore the salt solidifies more slowly than predicted by the analytical model to yield the heat which is extracted from the system. Also, the temperature gradient at the inner radius is less steep than predicted by the analytical model since less heat is conducted through the salt.

Figure 6 shows the wall temperature plotted vs. time for the axisymmetric system with several different heat extraction rates. Also shown in this figure are lines of constant radius of the liquid/solid interface.

To establish convergence for the enthalpy model of the axisymmetric case, it was necessary to find suitable values for the computer parameters ERR, NR, and NT. See Appendix C. It was found that when ERR = .00001, NR = 63, and NT = 80, the solution agreed within .002 of solutions with ERR = .000001, NR = 126, and NT = 160. In Figure 7 the approximate analytical solution, and the numerical solutions with the temperature formulation and enthalpy formulation are compared for a particular heat extraction rate and time. Agreement is seen to be excellent between the temperature formulation and the enthalpy formulation. Even the slight disagreement shown may be at least partially explained by the fact that the cannister wall is included in the enthalpy model but not in the temperature model.

Results with Fins

Because of the run time and memory requirements, the convergence criteria for the finned model was based on results with the axisymmetric model. The convergence study on the axisymmetric model showed that NT and ERR were most detrimental to accuracy. See Appendix D. Therefore the finned model cases were run with computer parameters NR = 21, NS = 30, NT = 80, and ERR = .00001.

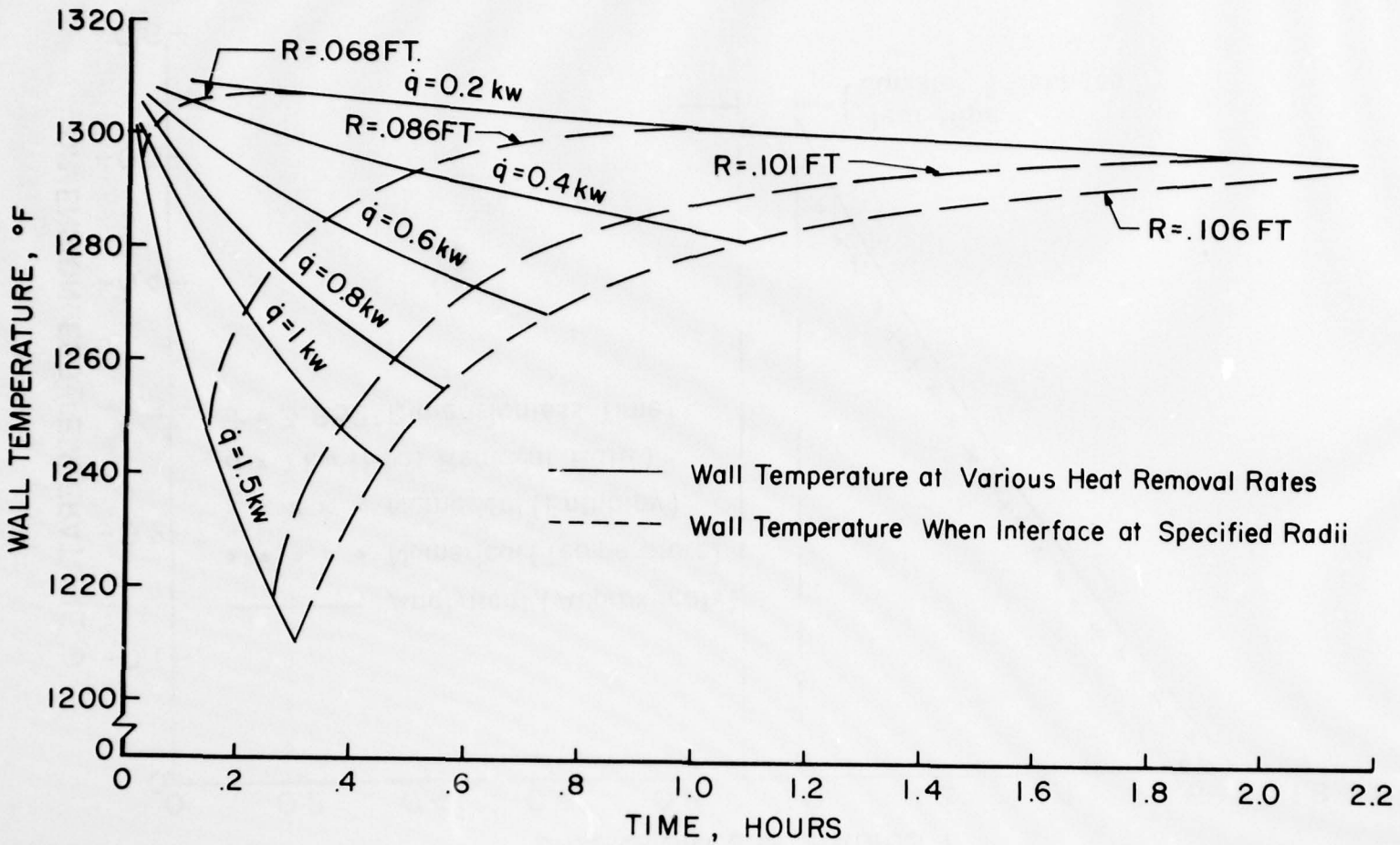

Figure 6. Wall Temperature and Interface Location in HP/TES System

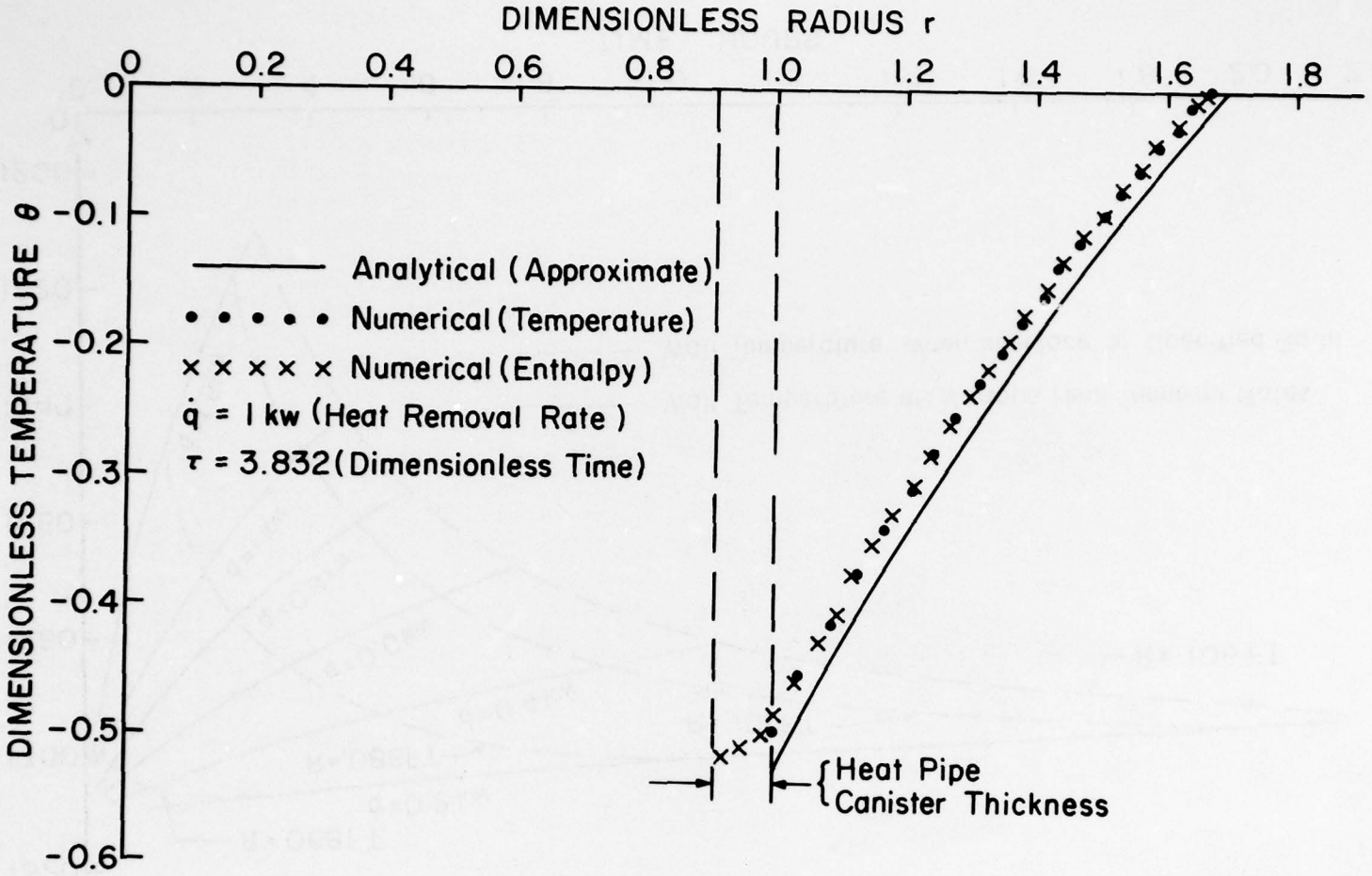

Figure 7. Temperature Distribution in Axisymmetric HP/TES System Determined Analytically and by Two Numerical Techniques

Figures 8, 10, 12, 14, and 16 show the interface radius plotted versus angle from the fin centerline for heat extraction rates of .25, .50, .75, 1.0, and 1.5 kw at several times. They show that a significant amount of freezing takes place along the fin but the close agreement of the interface shape for the two configurations indicates a similar amount of heat is extracted by the fins in each case. Figures 9, 11, 13, 15, and 17 show the wall temperature profiles for the same cases. Note that a significant reduction in the temperature drop is obtained with fins. This reduction is seen to increase with time and with heat extraction rate. It appears from the figures that most of the temperature drop obtained with 6 fins is also obtained with three fins.

Figure 18 shows the wall temperature as a function of the heat extracted for the zero, three and six fin configurations and a 1 kw heat rate. The effectiveness of fins as a device for reduction of total system weight was assessed in the following manner. The maximum allowable dimensionless temperature drop was arbitrarily selected to be 0.4. From Figure 18 the amount of heat extracted was as shown in column 2 of Table 2. The interface radius for the axisymmetric configuration was $\ell = 1.513$ and with a 30% void volume allowance, the outer radius is $r_o = 1.6857$ and the dimensionless volume is $v = 1.8416$. The other cases were sized to provide for the same volume of salt resulting in the outer radii shown in column 4 of the table. The lengths required to provide an equivalent amount of heat are shown in column 3 and the dimensionless system weights are shown in column 5. It may be seen that despite the shorter lengths for the finned configurations, the systems with fins are much heavier than the conventional configuration without fins. Thus the given fins are probably not an effective way to reduce the temperature drop when system weight is a prime consideration.

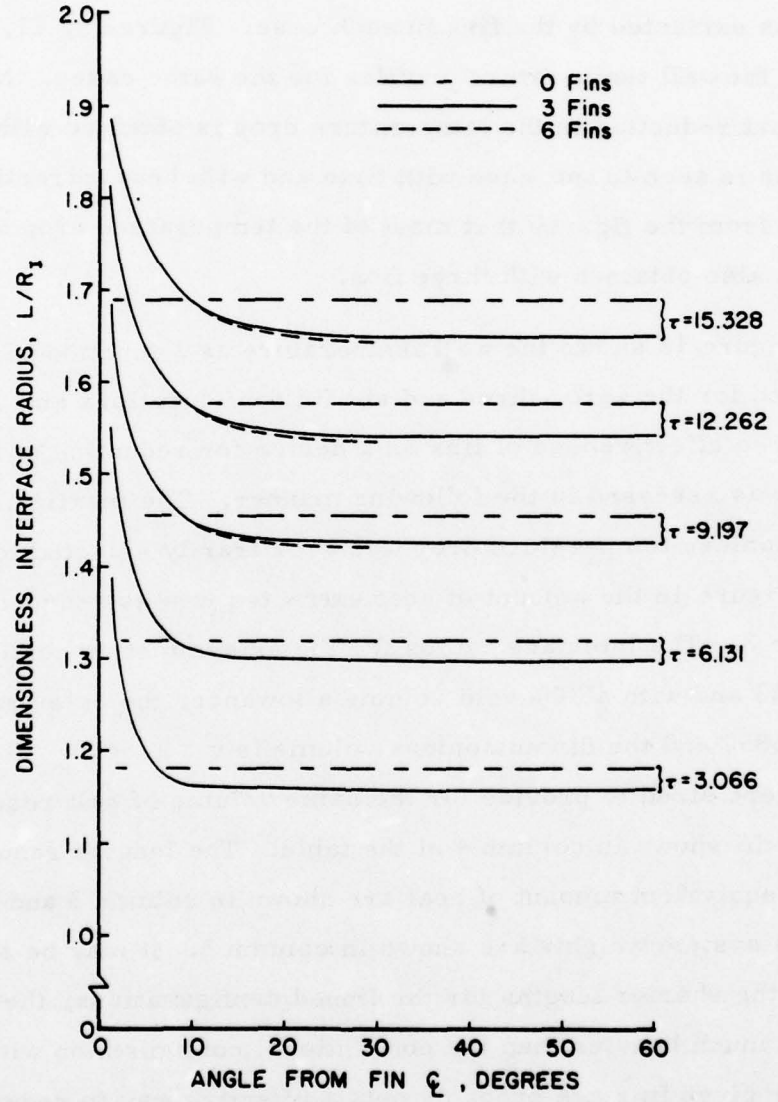

Figure 8. Interface Radius vs. Angle for 0, 3, & 6 Fin HP/TES Systems, Heat Extraction Rate = 0.25 kw

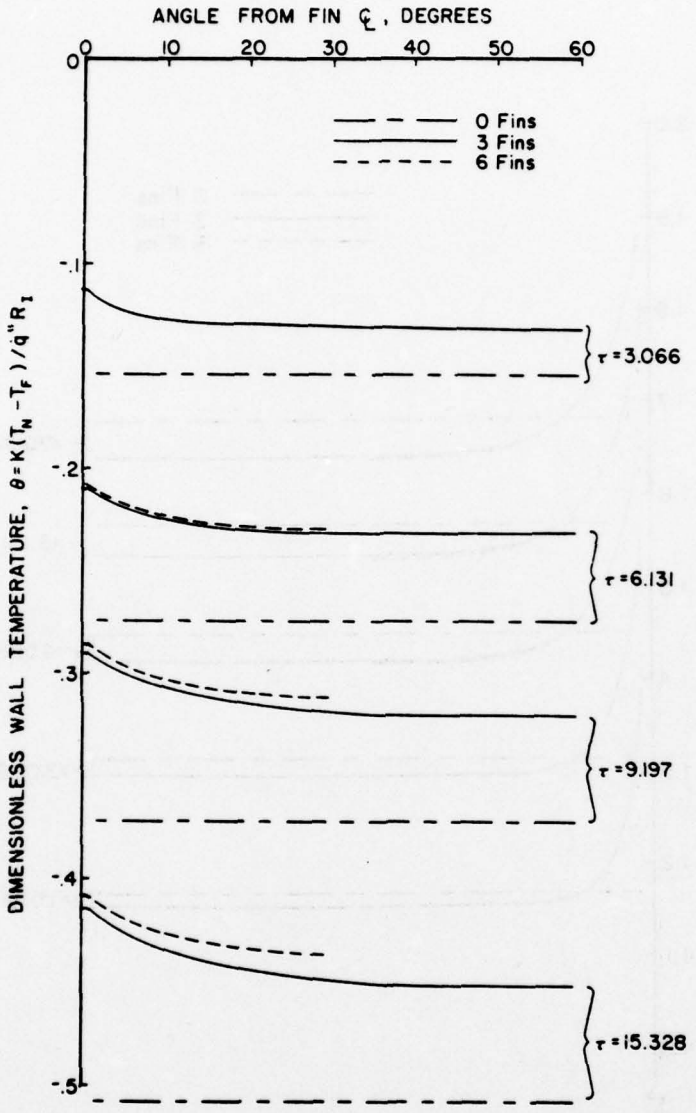

Figure 9. Wall Temperature vs. Angle for 0, 3, & 6 Fin HP/TES Systems, Heat Extraction Rate = 0.25 kw

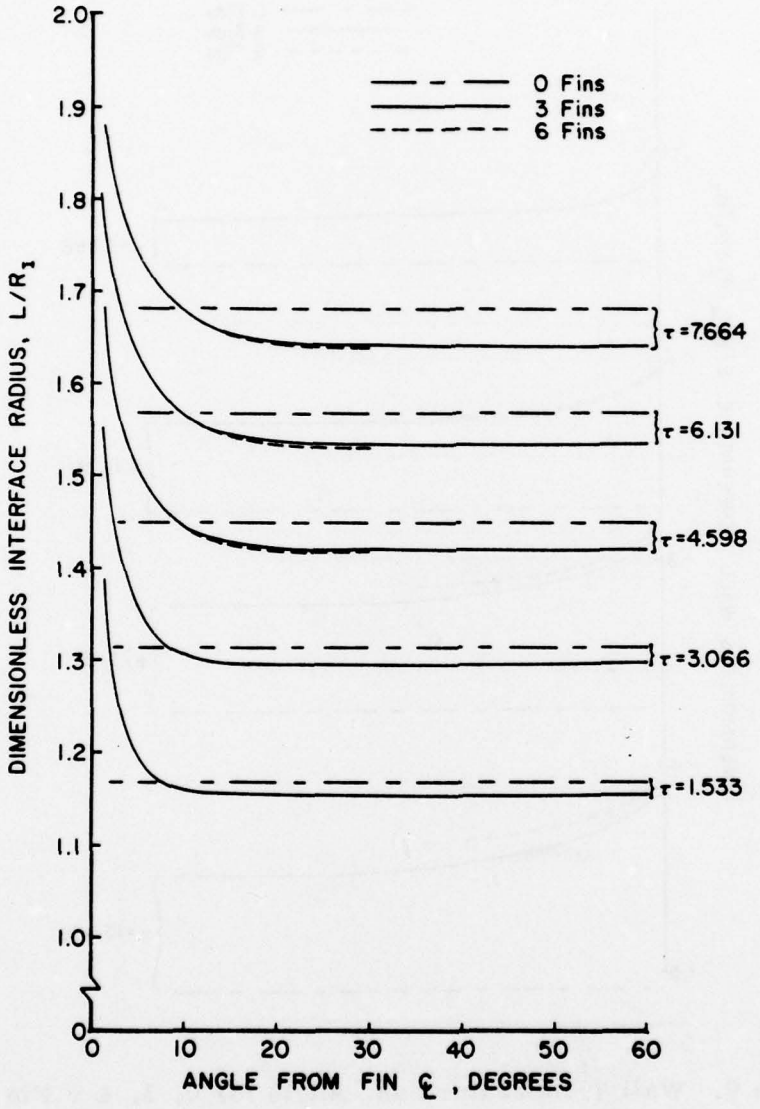

Figure 10. Interface Radius vs. Angle for 0, 3, & 6 Fin HP/TES Systems, Heat Extraction Rate = 0.50 kw

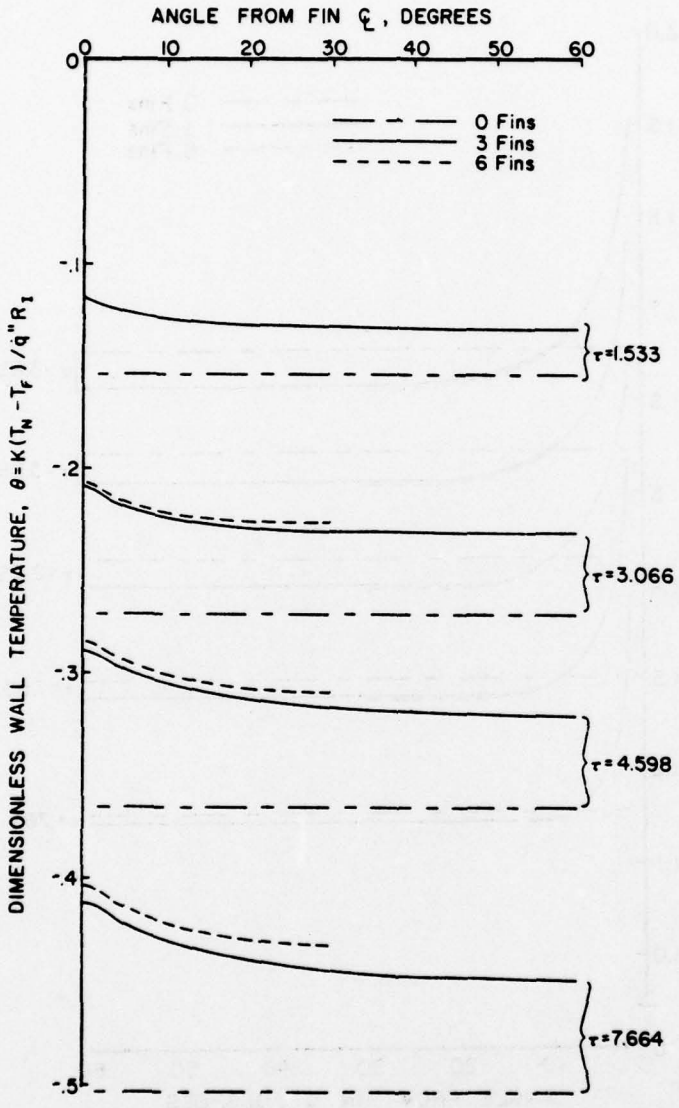

Figure 11. Wall Temperature vs. Angle for 0, 3, & 6 Fin HP/TES Systems, Heat Extraction Rate = 0.50 kw

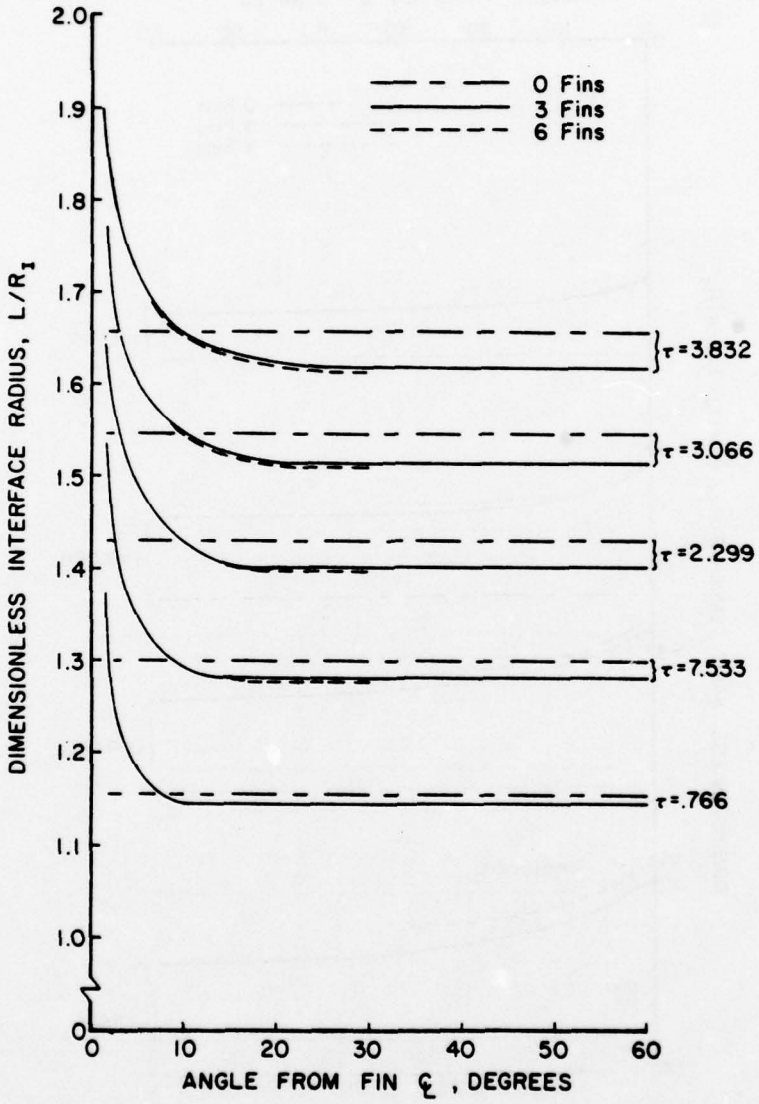

Figure 12. Interface Radius vs. Angle for 0 and 6 Fin HP/TES Units, Heat Extraction Rate = 0.75 kw

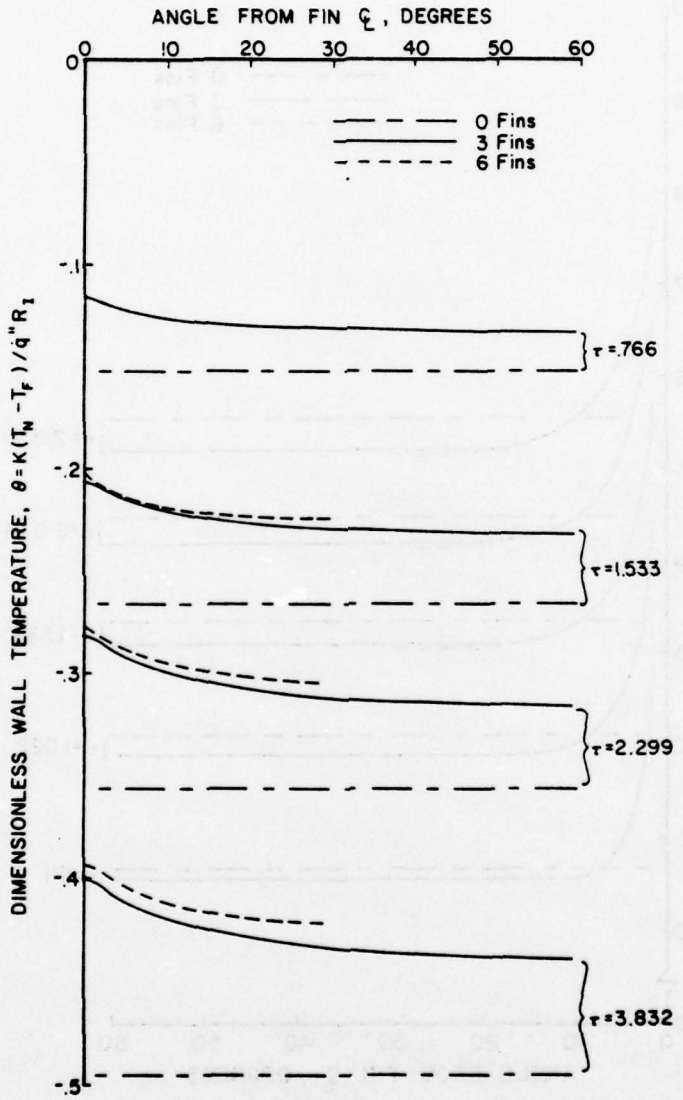

Figure 13. Wall Temperature vs. Angle for 0 and 6 Fin HP/TES Units, Heat Extraction Rate = 0.75 kw

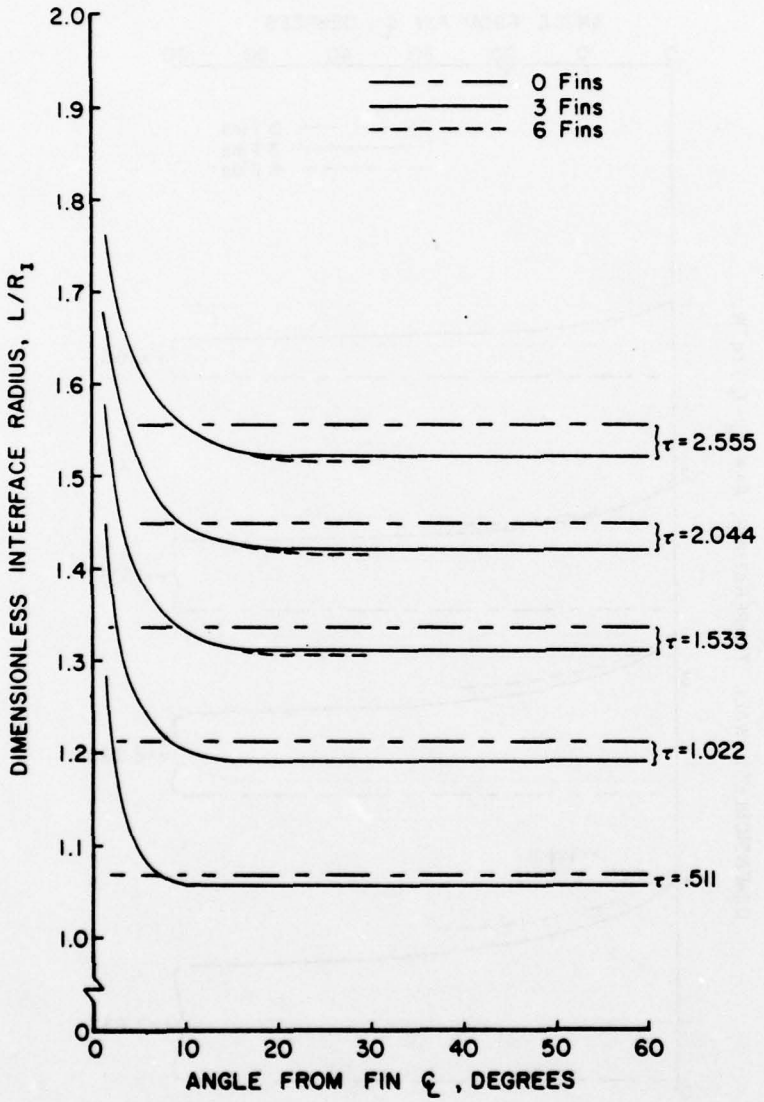

Figure 14. Interface Radius vs. Angle for 0, 3, & 6 Fin HP/TES Units, Heat Extraction Rate = 1.00 kw

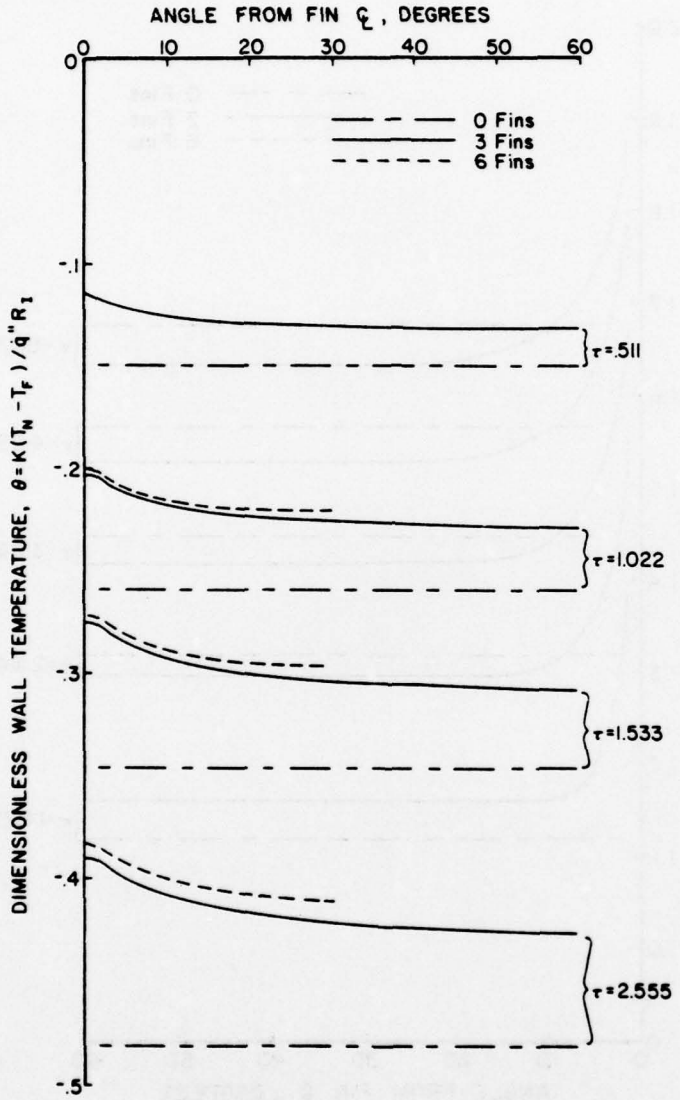

Figure 15. Wall Temperature vs. Angle for 0, 3, & 6 Fin HP/TES Units, Heat Extraction Rate = 1.00 kw

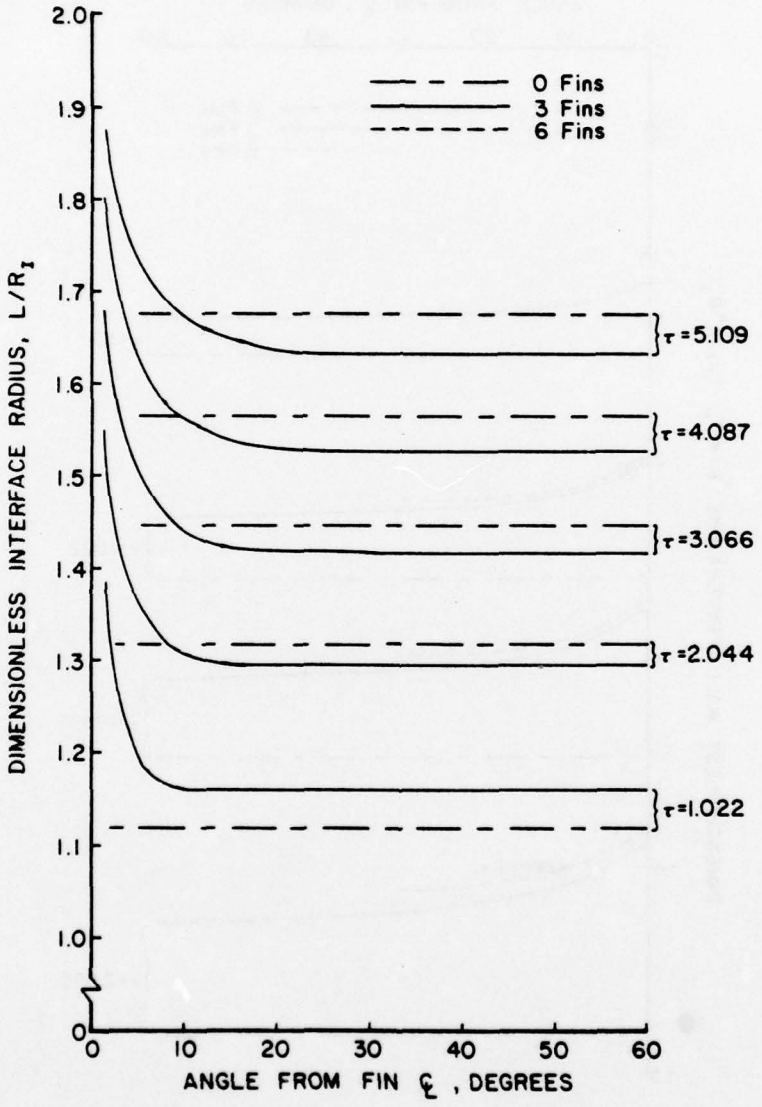

Figure 16. Interface Radius vs. Angle for 0, 3, & 6 Fin HP/TES Units, Heat Extraction Rate = 1.50 kw

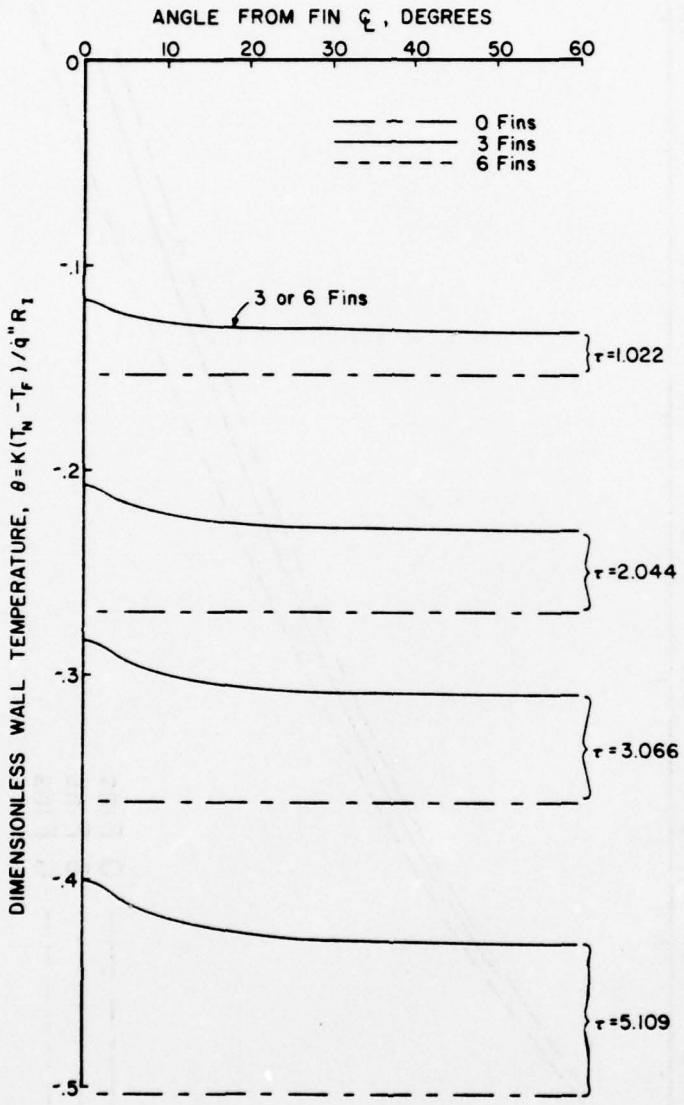

Figure 17. Wall Temperature vs. Angle for 0, 3, & 6 Fin HP/TES Units, Heat Extraction Rate = 1.50 kw

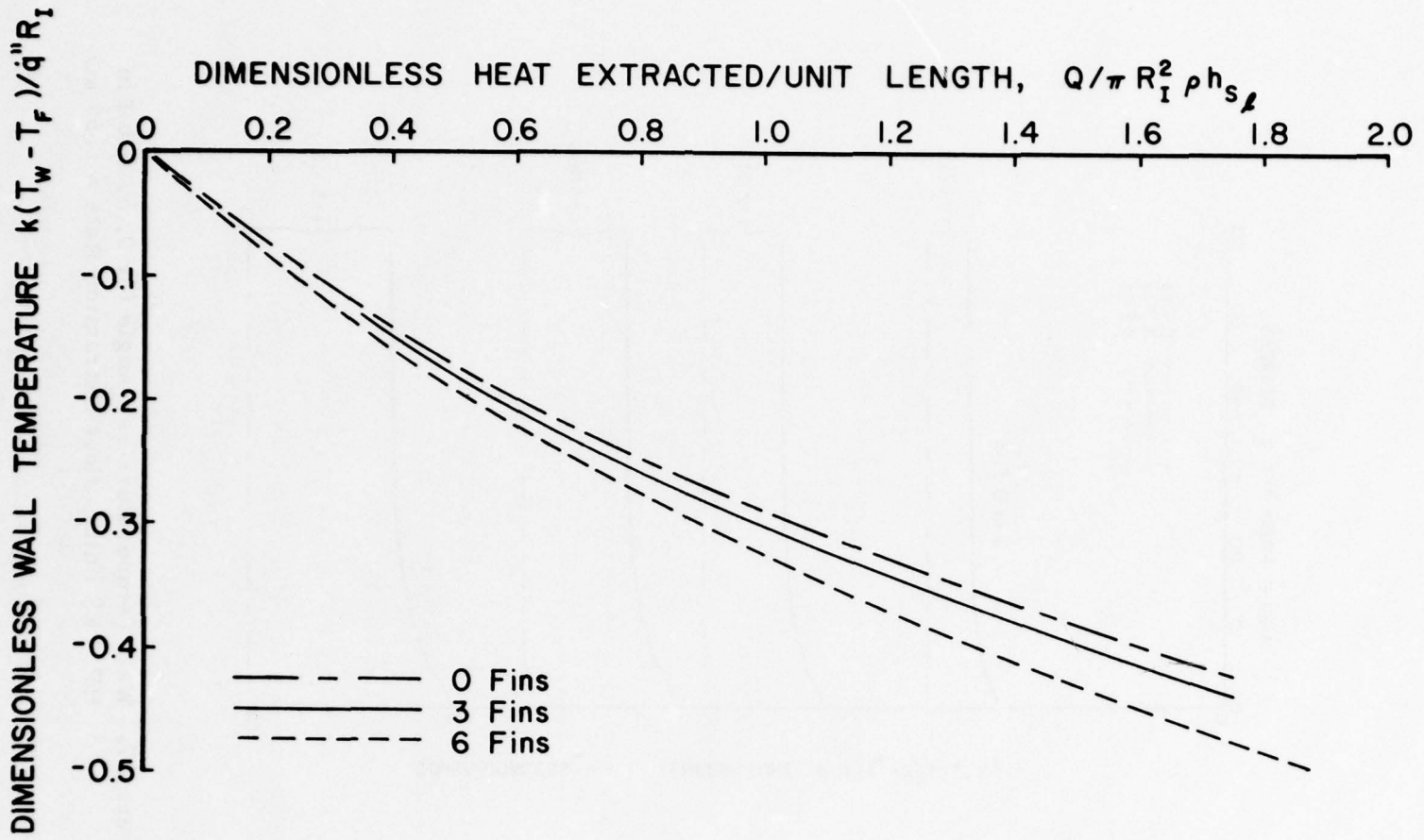

Figure 18. Heat Extracted vs. Wall Temperature for 0, 3, & 6 Fin HP/TES Units

TABLE 2

FIN EFFECTIVENESS

Heat Extraction Rate = 1 kw
Dimensionless Wall Temperature = -0.4

No. of Fins	Heat Extracted / Unit Length	Relative Length	R_0/R_I	Relative Weight
0	1.36	1.0	1.686	1.
3	1.52	.8947	1.766	1.0387
6	1.62	.8395	1.823	1.1189

SECTION VIII
SUMMARY AND CONCLUSIONS

The heat transfer process has been successfully modeled for both axisymmetric and finned systems. For axisymmetric systems the temperature formulation may be solved very quickly, providing a solution which is extremely accurate. The difficulty in locating the interface boundary makes it difficult to adapt this formulation to two dimensional systems.

For those systems, the enthalpy model provides a system in which the temperature distribution and interface boundary are found simultaneously. When this formulation was applied to the axisymmetric system it was found that accuracies were as good as those attained with the temperature model but that an order of magnitude greater computer time was required to obtain the solution.

For the finned system, an additional order of magnitude of computer time was required to obtain the solution. The solutions with fins are slightly less accurate than the axisymmetric since it wasn't practical to study convergence fully. Solutions were attained with sufficient accuracy, however, to give a meaningful comparison of the finned systems to the axisymmetric system. It was found that some reduction of the temperature drop at the inner wall was attained with fins but that systems employing fins would in turn be heavier. Thus fins may not be practical for this particular fin/system design.

SECTION IX
RECOMMENDATIONS

Although the analytical and numerical techniques developed were very accurate, the final solution may be adversely affected by the accuracy of the measured properties of the salts as well as the limiting assumptions used in deriving the governing equations. Based on the analysis completed it is recommended that

1) The salt physical properties, liquid and solid, should be accurately determined.
2) The effect of radiation to and from the salt, liquid and solid, should be assessed. Radiation properties of the salt are needed.
3) The experimental apparatus should be redesigned to (a) relieve hydrostatic stresses on melting, (b) improve calorimetry and (c) determine a fin configuration to optimize heat transfer.
4) The temperature model for the axisymmetric case may be useful to study boundary conditions which vary with time since it is solved so quickly. It is recommended that this program be suitably modified and studies of the effects of time varying heat extraction rate be conducted.
5) The enthalpy model has the versatility to study solidification fronts in unusual geometries. This model should be refined to shorten the solution time and better define the accuracy. It could then be used to examine other two dimensional configurations, including the axial variation of the nonfinned case and the assymmetry due to the volume void on solidification.
6) The model of the axisymmetric system should be expanded to include the heat transfer within the heat pipe and the calorimeter.

Such an expanded model would be useful in the design of further experiments with such systems.

APPENDIX A

AXISYMMETRIC HEAT CONDUCTION EQUATION

In this appendix the axisymmetric 1-Dimensional heat conduction equation in cylindrical coordinates is written in finite difference form for a variable mesh grid. This equation is:

$$\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} = \frac{\partial\theta}{\partial\tau} \tag{A.1}$$

Where θ is the dimensionless temperature at time τ and radius r. The object is to express Equation (A.1) at $r = r_k$ in terms of the temperatures θ_{k-1}, θ_k and θ_{k+1} at radii r_{k-1}, r_k, and r_{k+1} respectively. To do this write θ_{k+1} and θ_{k-1} by Taylor Series:

$$\theta_{k+1} = \theta_k + (r_{k+1}-r_k)\frac{\partial\theta}{\partial r} + \frac{(r_{k+1}-r_k)^2}{2!}\frac{\partial^2\theta}{\partial r^2} + \cdots$$

$$\theta_{k-1} = \theta_k - (r_k-r_{k-1})\frac{\partial\theta}{\partial r} + \frac{(r_k-r_{k-1})^2}{2!}\frac{\partial^2\theta}{\partial r^2} + \cdots \tag{A.2}$$

Neglect higher order terms and solve Equations (A.2) for $\frac{\partial\theta}{\partial r}$ and $\frac{\partial^2\theta}{\partial r^2}$ to obtain

$$\frac{\partial\theta}{\partial r} \approx \frac{(r_k-r_{k-1})^2\,\theta_{k+1} + (-r_{k+1}+2r_k-r_{k-1})(r_{k+1}-r_{k-1})\,\theta_k - (r_{k+1}-r_k)^2\,\theta_{k-1}}{(r_{k+1}-r_k)(r_k-r_{k-1})(r_{k+1}-r_{k-1})}$$

$$\frac{\partial^2\theta}{\partial r^2} \approx 2\,\frac{(r_k-r_{k-1})\,\theta_{k+1} - (r_{k+1}-r_{k-1})\,\theta_k + (r_{k+1}-r_k)\,\theta_{k-1}}{(r_{k+1}-r_k)(r_k-r_{k-1})(r_{k+1}-r_{k-1})} \tag{A.3}$$

Substitute Equation (A.3) into the right hand side of (A.1) and group terms to obtain:

$$\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} = \frac{1}{r_k(r_{k+1}-r_{k-1})(r_{k+1}-r_k)(r_k-r_{k-1})}$$

$$\{(3r_k-r_{k+1})(r_{k+1}-r_k)\theta_{k-1} + (4r_k-r_{k+1}-r_{k-1})(r_{k+1}-r_{k-1})\theta_k$$

$$+ (3r_k-r_{k-1})(r_k-r_{k-1})\theta_{k+1}\}$$

$$= -2(c_k\theta_{k-1}+a_k^1\theta_k+b_k\theta_{k+1}) \qquad (A.4)$$

Let θ represent the temperature at time τ and ϕ represent the temperature at an earlier time $\tau-\Delta\tau$. The right hand side of Equation (A.1) is written by finite differences and the Crank-Nicholson method is employed. Then:

$$\frac{1}{2}\left\{\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r}\right\} = c_k(\theta_{k-1}+\phi_{k-1})$$

$$+ a_k'(\theta_k+\phi_k) + b_k(\theta_{k+1}+\phi_{k+1}) = \frac{\partial\theta}{\partial\tau} = \frac{\theta_k-\phi_k}{\Delta\tau} \qquad (A.5)$$

Thus, grouping the unknown terms on the left hand side the finite difference form of Equation (A.1) is

$$c_k\theta_{k-1} + a_k\theta_k + b_k\theta_{k+1} = d_k \qquad (A.6)$$

where

$$c_k = -\frac{(3r_k-r_{k+1})(r_{k+1}-r_k)}{2r_k(r_{k+1}-r_{k-1})(r_{k+1}-r_k)(r_k-r_{k-1})}$$

$$b_k = -\frac{(3r_k-r_{k-1})(r_k-r_{k-1})}{2r_k(r_{k+1}-r_{k-1})(r_{k+1}-r_{k)})(r_k-r_{k-1})}$$

$$a_k = -\frac{(4r_k - r_{k-1} - r_{k+1})(r_{k+1} - r_{k-1})}{2r_k(r_{k+1} - r_{k-1})(r_{k+1} - r_k)(r_k - r_{k-1})} + \frac{1}{\Delta\tau}$$

$$d_k = -c_k\phi_{k-1} + (-a_k + \frac{2}{\Delta\tau})\phi_k - b_k\phi_{k+1}$$

If ϕ_k is known for k = 1, 2, . . . , n where r_n is the radius of the interfacial surface at time $\tau - \Delta\tau$, Equation (A.6) is applicable for k = 2, 3, . . . , n-1. At the inner boundary an imaginary node is assumed such that

$$r_2 - r_1 = r_1 - r_0 \tag{A.7}$$

and the inner boundary condition is expressed in finite difference form:

$$\frac{\phi_2 - \phi_0}{2(r_2 - r_1)} = \frac{\theta_2 - \theta_0}{2(r_2 - r_1)} = 1. = \frac{\partial\theta}{\partial r} \tag{A.8}$$

substitution of Equations (A.7) and (A.8) into Equation (A.6) for k = 1 yields the equation for the inner boundary:

$$\left\{\frac{1}{(r_2 - r_1)^2} + \frac{1}{\Delta\tau}\right\}\theta_1 - \frac{1}{(r_2 - r_1)^2}\theta_2 = \frac{1}{(r_2 - r_1)^2}(\phi_2 - \phi_1) + 1. - \frac{2.}{r_2 - r_1} + \frac{\phi_1}{r_2 - r_1} \tag{A.9}$$

For k = n Equation (A.4) is not valid for ϕ since $\frac{\partial\phi}{\partial r}$ is not defined at r_n. Instead, from Equation (A.1)

$$\frac{\partial^2\phi_n}{\partial r^2} + \frac{1}{r}\frac{\partial\phi_n}{\partial r} = \frac{\partial\phi_n}{\partial\tau} = \frac{\partial\phi_n}{\partial r}\frac{\partial r_n}{\partial\tau} = \frac{\partial\phi_n}{\partial r}\frac{\partial\ell}{\partial\tau}$$

and from Equation (15)

$$\frac{\partial\phi_n}{\partial r} = \frac{1}{2S_{T_e}}\frac{\partial\ell}{\partial\tau}$$

so that, in finite difference form

$$\frac{\partial^2 \phi_n}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_n}{\partial r} = \left(\frac{\ell - r_n}{\tau_{n+1} - \tau_n} \right)^2 / 2 \, S_{T_e} \tag{A.7}$$

and substitution into Equation (A.5) yields

$$c_n \theta_{n-1} + a_n \theta_n = \frac{(\ell - r_n)^2}{(\tau_{n+1} - \tau_n)^2} / 2 \, S_{T_e} + \frac{\phi_n}{\Delta \tau} \tag{A.8}$$

where c_n and a_n are given by Equation (A.6) with ℓ used for r_{k+1}.

APPENDIX B

PROGRAM ICE2

This appendix documents the program ICE2 which finds the temperature distribution and interface radius for the axisymmetric system using the temperature formulation developed in Section V.2.b. Figure B-1 is a program listing. Figure B-2 shows the necessary control cards and sample input data to run the program. The input is organized as follows:

Record 1 has a format of 5E12.0 and contains the following variables

DIF - diffusivity of thermal storage material, ft^2/hr

CON - thermal conductivity of thermal storage material, BTU/ft-hr-^oF

DEN - density of thermal storage material, lbm/ft^3

TF - fusion temperature of thermal storage material, ^oF

HSL - latent heat of fusion of thermal storage material, BTU/lbm

Record 2 has a format of 5E12.0 and contains the following variables

RI - outer radius of heat pipe wall, ft

RO - inner radius of outer canister wall, ft

QR - heat flux rate at outer heat pipe wall, BTU/ft^2hr

DNR - ratio of solid salt volume to canister volume

DT - time between successive calculations, $\frac{\alpha_f \Delta t}{R_I^2}$

Record 3 has a format of 4E12.0 and contains the following variables

DFF - canister wall diffusivity, ft^2/hr

CFN - canister wall conductivity, BTU/ft-hr-^oF

DFN - canister wall density, lbm/ft^3

P - canister wall thickness, ft

Record 4 has a format of E12.0, 3I5 and contains the following variables

EPS - tolerance on calculation of energy integral

IP - print every IPth time calculations are made

L - if L = 1 output is dimensionless otherwise it is dimensional

ICN - case number for identification

```
PROGRAM ICE2        74/74   OPT=1                                   FTN 4.6+446

      PROGRAM ICE2(INPUT,OUTPUT,TAPE2=OUTPUT)
      DIMENSION S(100)
      COMMON RT(100),TNT,B(100),SUB(100),SUP(100),D(100),RH(100),R(100),
     1F
      READ 100, DIF,CON,DEN,TF,HSL
      READ 100, RI,RG,GR,DNR,DT
      READ 100,DFF,OFN,DFN,F
      READ 500, EPS,IP,L,ICN
      DO 40 I=1,50
      B(I)=0.
40    RT(I)=0.
      IC=IP
      RMAX=SQRT(DNR*RG*FO+(1.-DNR)*RI*RI)/RI
      TR=GR*RI/CON
      F=QR*RI/DIF/DEN/HSL
      TNT=DT
      DELT=TF*ALOG(RI/(RI-F))
      R(1)=1.
      S(1)=RI
      RG=1.
      L=3.*F/(4.+2.*F)
      WRITE(2,550)
      IF(L.EQ.1)GO TO 55
      WRITE(2,650)ICN
      GO TO 65
55    WRITE(2,600)ICN
65    DNR=DNR*100
      WRITE(2,700)QR,DNR,TR,TF,RI,RG,F
      WRITE(2,750)DEN,CON,DIF,HSL,DFN,OFN,DFF
      WRITE(2,800)DELT
      DNR=DNR/100
      DO 50 I=1,50
      R(2)=SQRT(1.+2.*(DT+ALOG(RG)/2.))
      IF (ABS(R(2)-RG).LT.1.E-6*RG) GO TO 60
      RG=R(2)
50    CONTINUE
60    CONTINUE
      S(2)=R(2)*RI
      RT(1)=ALOG(R(1)/R(2))
      B(1)=1./(R(2)-R(1))/(R(2)-R(1))
      SUB(1)=-B(1)
      SUP(1)=B(1)*(RT(2)-RT(1))+1.-2./(R(2)-R(1))
      I=2
      DNRG=(1.-R(2)*R(2))/4.+ALOG(R(2))/2.
      DNRG=DNRG
      TF=TF+DT
10    YP=SQRT(1.+2.*F*(TF+DNRG))
      CALL DFUN(YP,TF,NT,DNRG)
      IF (ABS(DNRG-DNRG).LT.-DNRG*EPS) GO TO 70
      DNRG=DNRG
      GO TO 10
```

Figure B-1. Program ICE2

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```
   70   IF (YP.LT.(MAX+.01*(YP-R(NT+1))) GO TO 80
        TP=TMT+(SM/X-R(NT))/(YP-R(NT))*(TF-TMT)
        XO=TP
        GO TO 10
   80   R(NT+1)=YF
        S(NT+1)=R(NT+1)*R1
        DO 20 I=2,NT
   20   BH(I)=-(SUB(I)*B(I-1)+D(I)*B(I)+SUP(I)*B(I+1))
        NT=NT+1
        DO 30 I=1,NT
   30   BT(I)=B(I)
        IF (XO.LT.TP) GO TO 50
        DIF=TP-XO+I/DIF
        IF(L.EQ.1) GO TO 35
        WRITE (2,200) TM,S(NT)
        WRITE (2,400)
   15   DO 5 I=1,NT
    5   B(I)=TF+TM*B(I)
        IF(L.EQ.1) GO TO 45
        WRITE (2,300) (S(I),B(I),I=1,NT)
   25   IC=0
        IF (TF.LT.2.01*DT) IC=2
   50   IC=IC+1
        TF=TF+DT
        TM=TMT+DT
        IF(YF.LT.(MAX-.01*(R(NT)-R(NT-1))) GO TO 10
  100   FORMAT(5E12.0)
  200   FORMAT (6H TIME=,F15.6,2HHR,/," RADIUS OF LIQUID/SOLID INTERFACE=
       1,E15.6,2HFT)
  250   FORMAT ( //,6H TIME=,F8.5,22X,17HWALL TEMPERATURE=,
       1F8.7,/,1H ,11X,*RADIUS OF LIQUID/SOLID INTERFACE=*,F9.6)
  300   FORMAT (15(1H ,F8.4,F8.2,F8.4,F8.2,F8.4,F8.2,F8.4,F8.2,/))
  350   FORMAT(15(1H ,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,F8.4,/))
  400   FORMAT (1H ,4(16H  RAD,FT  TEMP,F))
  450   FORMAT(1H0,4(16H    RAD      TEMP ))
  500   FORMAT (F12.0,3I5)
  550   FORMAT (1H1,5X,*HEAT TRANSFER IN PHASE CHANGE/THERMAL ENERGY STO
       1GE SYSTEM*)
  600   FORMAT (//,1H ,*SYMMETRIC TEMPERATURE FORMULATION-CASE NO*,I3,*
       1  DIMENSIONLESS OUTPUT *)
  650   FORMAT (//,1H,* SYMETRIC TEMPERATURE FORMULATION-CASE NO*,I3,*
       1  DIMENSIONAL OUTPUT*)
  700   FORMAT (1H0,*  HEAT FLUX RATE=*,F10.3," BTU/FT**2-HR      FILLED V
       1OLUME",F6.2,* PERCENT*,/,1H0,*  DIFFERENCE TEMPERATURE=*,F7.2,* F  I
       1NITIAL/BOUNDARY TEMPERATURE=*,F5.2,2H F,///,1H ,26X,*CANNISTER GEOME
       1TRY*,//,1H ,22X,*INNER RADIUS=*,F10.6,3H FT,//,1H ,22X,
       1*OUTER  RADIUS=*,F10.6,3H FT,/,1H0,20X,*WALL THICKNESS=*,F10.6,
       13H FT,///,1H ,17X,*THERMAL STORAGE MATERIALS PROPERTIES*)
  750   FORMAT ( /,1H ,27X,*DENSITY=*,F10.4," LBM/FT**3",/,1H0,14X,*THERM
       1AL CONDUCTIVITY=*,F10.6,* BTU/HR-FT-F*,/,1H0,15X,*THERMAL DIFFUSIV
       1ITY=*,F10.7," FT**2/HR",/,1H0,13X,*LATENT HEAT OF FUSION=*,F10.4,*
       1 BTU/LBM*,///,1H ,20X,*CANNISTER MATERIAL PROPERTIES*,//,1H ,27X,*
       1DENSITY=*,F10.4," LBM/FT**3",/,1H0,14X,*THERMAL CONDUCTIVITY=*,F10
       1.6,* BTU/HR-FT-F*,/,1H0,15X,*THERMAL DIFFUSIVITY=*,F10.7," FT**2/H
       1R")
```

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```
C00     FO RMAT (///,1H ,10X,*PEAK PRESSURE OF ACCESS CAMPSITE WALL=*,F8.1
        1,* PSI*,/,1H1)
        STOP
35      WRITE(2,250) TP,PT(1),T(NT)
        WRITE(2,450)
        GO TO 15
45      WRITE(2,350) (T(I),PT(I),I=1,NT)
        GO TO 25

        END




        SUBROUTINE DFUN(YP,TP,NT,DF,RG)
        DIMENSION PT(100)
        COMMON PT(100),T(100),B(100),SUB(100),SUP(100),D(100),RH(100),R(100),
       1P
        RG=P(NT)
        RL=P(NT-1)
        BT=.5/(RG*(YP-RG)*(RG-RL)*(YP-RL))
        SUB(NT)=-BT*(YP-RG)*(3.*RG-YP)
        D(NT)=BT*(YP-RL)*(4.*RG-YP-RL)
        SUP(NT)=-BT*(RG-RL)*(3.*RG-RL)
        RH(NT)=((YP-RG)/(TP-TNT))**2/2./P
        DO 10 I=1,NT
        DI (I)=D(I)+1./(TP-TNT)
10      B(I)=RH(I)*PT(I)/(TP-TNT)
        CALL TRIL(SUB,DIA,SUP,B,NT)
        DNRG=0.
        DO 20 I=2,NT
20      DNRG=DNRG+(B(I)*R(I)+B(I-1)*R(I-1))*(R(I)-R(I-1))/2.
        DNRG=DNRG+P(NT)*R(NT)*(YP-P(NT))/2.
        RETURN
        END




        SUBROUTINE TRIL(SUB,DIA,SUP,B,N)
        DIMENSION SUB(100),DIA(100),SUP(100),B(100)
        IF(N.GT.1) GO TO 10
        B(1)=B(1)/DIA(1)
        RETURN
10      DO 20 K=2,N
        RAT=-SUB(K)/DIA(K-1)
        DIA(K)=DIA(K)+RAT*SUP(K-1)
20      B(K)=B(K)+RAT*B(K-1)
        B(N)=B(N)/DIA(N)
        K=N
        DO 30 KD=2,N
        K=K-1
30      B(K)=(B(K)-SUP(K)*B(K+1))/DIA(K)
        RETURN
        END
```


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```
UD121,T15,IO10,CM60000,STCSA.F740579,BANDOW,KL121,229-2835
COMMENT.**********NO DECK**********
COMMENT.**********93392**********
ATTACH,F,ICE,CY=1.
FTN,I=F,L,R=3,B=FLGO.
RETURN,F.
REWIND,FLGO.
FLGO,FL=20000.
*EOR
    .035          4.11          181.          1310.          350.
   .0625        .1195833      8691.133      .7056356    .095800525
  .2244971        13.5          501.12      .005416667
   .0001          1     1      0
*EOR
*EOF
```

Figure B-2

Control Cards

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APPENDIX C

PROGRAM ENTH

This appendix documents the program ENTH which calculates the axisymmetric temperature distribution and interface radius from the enthalpy formulation developed in Section VI.1.a. Figure C-1 is a program listing, Figure C-2 shows the control cards and sample input data. The input is organized as follows:

Record 1, format 5E12.0

DIF - thermal diffusivity of thermal storage material, ft^2/hr
CON - thermal conductivity of thermal storage material, $BTU/ft\text{-}hr\text{-}^oF$
DEN - density of thermal storage material, lbm/ft^3
TF - fusion temperature of thermal storage material, ^oF
HSL - latent heat of fusion of thermal storage material, BTU/lbm

Record 2, format 5E12.0

RI - outer radius of heat pipe wall, ft
RO - inner radius of outer canister wall, ft
QR - heat flux rate at outer edge of heat pipe, BTU/ft^2hr
DNR - ratio of solid thermal storage material volume to canister volume
ERR - allowable error in enthalpy change

Record 3, format 4E12.0

ORF - relaxation factor effects speed of convergence
DFF - thermal diffusivity of canister wall, ft^2/hr
CFN - conductivity of canister wall, $BTU/ft\text{-}hr\text{-}^oF$
DFN - density of canister wall, lbm/ft^3

Record 4, format 7I5

NR - number of nodes in radial direction in salt
NT - number of points in time

IM - maximum number of iterations allowed

IP - print every IPth time

IR - dummy variable

IF - number of nodes in radial direction in wall

ICN - case number for identification

```
JOB=ENTH      74/74   OPT=1                                  FTN 4.6+446

      PROGRAM ENTH(INPUT,TAPE1=INPUT,OUTPUT,TAPE2=OUTPUT)
      DIMENSION H(250),T(250),C(250),PH(250),P(250)
      READ (1,100) DIF,CON,DEN,TF,HSL
      READ (1,100) RI,RO,QR,DNR,ERR
      READ (1,100) OFF,DFF,CFN,DFN
      READ (1,200) NO,NT,IM,IF,IN,IT,IGN
      IC=IF
      RMAX=SQRT((QNR*QR*RO+(1-QNR)*QR*RI)/RI
      DR=(RO-RI)/RI/NF
      ST=QR*RI/DIF/DEN/HSL
      DT=(RMAX*TMAX-1.)/2./ST/NT
      QF=DIF*QFM/DFF/COM
      TREF=QR*RI/CON
      P=IF*QR*RI
      DO 10 I=1,250
      R(I)=(I-IF-.5)*DR+1.
      C(I)=QF/R(I)**2
      PH(I)=1.
      IF (I.LE.IF) PH(I)=0.
      H(I)=PH(I)
   10 T(I)=0.
      IS=0
      D=DEN*DR/DT
      QN=QNR*100
      WRITE (2,300) IGN,QR,QNR,TREF,TF,RI,RO,P
      WRITE (2,350) DEN,CON,DIF,HSL,DEN,QFM,DFF
      WRITE (2,450) NO,NT,IM,ERR,ORF
      QN=QNR/100
   30 IT=0
   50 OH=H(1)
      DF=1.
      QN1=1.
      QN2=1.
      IF (1.LE.IF) DF=DFF
      IF (1.LT.IF) QN1=CFN/CON
      IF (1.LE.IF) QN2=CFN/CON
      IF (1.EQ.IF) QN1=2.*CON*CFN/(CON+CFN)/CON
      IF (1.EQ.IF+1) QN2=2.*CON*CFN/(CON+CFN)/CON
      H(1)=(D*PH(1)+QN1*(1.+C(1))**2)-2.*C(1)*ST)/(D+QN1/DF*(1.+C(1)))
      IF (QH.LT.0.) H(1)=DFF*H(1)+(1.-DRF)*QH
      T(1)=H(1)/DF
      ER=ABS(H(1)-QH)
      EQ=ABS(H(1))
      DO 20 I=2,250
      NN=I
      QH=H(I)
      DF=1.
      QN1=1.
      QN2=1.
```

Figure C-1. Program ENTH

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```
      IF (I.LE.IF) DF=DFR
      IF (I.LT.IF) CN1=CFN/CCN
      IF (I.LE.IF) CN2=CFN/CGN
      IF (I.EQ.IF) CN1=2.*CCN*CFN/(CCN+CFN)/CCN
      IF (I.EQ.IF+1) CN2=2.*CGN*CFN/(CGN+CFN)/CGN
      H(I)=(D*PH(I)+(1.+C(I))*(CN1*T(I+1)+(1.-C(I))*CN2*T(I-1))/(D+(CN1*
     1(1.+C(I))+CN2*(1.-C(I)))/DF)
      IF (H(I).LT.0.) GO TO 30
      H(I)=H(I)*(D+(CN1*(1.+C(I))+CN2*(1.-C(I)))/DF)/D
      T(I)=0.
      GO TO 40
   30 CONTINUE
      IF (OH.LT.0.) H(I)=ORF*H(I)+(1.-ORF)*OH
      T(I)=H(I)/CF
      EN=EN+ABS(H(I)-OH)
      ED=ED+ABS(H(I))
   20 CONTINUE
   40 CONTINUE
      IF (EN.LT.ED*ENF) GO TO 60
      IT=IT+1
      IF (IT.LT.IM) GO TO 50
   60 IS=IS+1
      IC=IC+1
      IF (IC.LT.IF) GO TO 5
      NM1=NM-1
      DF=DT*IS
      F=(NM-IF-F(NM))*DR+1.
      W1=(CGN*T(IF+1)+CFN*T(IF))/(CGN+CFN)/ST
      TW=(3.*T(1)-T(2))/2./ST
      SM=0.
      HE=0.
      DO 15 I=1,NM
      HI=1.
      IF (I.LE.IF) HI=0.
      HJ=AMAX1(0.,H(I))
      SM=SM+2.*R(I)*DR*(HI-HJ)
      HE=HE+2.*R(I)*DR*(HI-H(I))
   15 PH(I)=T(I)/ST
      FO=EN/ED
      WRITE (2,400) TF,HE,SM,IT,FO,TW,TW1,RF
      WRITE (2,500) (R(I),PH(I),I=1,NM1)
      IC=0
      IF (IS.LE.1) IC=1
    5 DO 70 I=1,NM
   70 PH(I)=H(I)
      IF (IS.LT.MT) GO TO 80
  100 FORMAT (5E12.0)
  200 FORMAT (7I5)
  300 FORMAT (1H1, 6X,'HEAT TRANSFER IN PHASE CHANGE/THERMAL ENERGY STO
     1AGE SYSTEM',///,1H ,'SYMMETRIC ENTHALPY FORMULATION-CASE NO.
     1',I3,4X,'DIMENSIONLESS OUTPUT',/,1H0,'HEAT FLUX RATE=',F10.3,' BT
     1U/FT**2-HR',5X,'FILLED VOLUME=',F5.1,' PERCENT',/,1H0,'REFERENCE
     1TEMPERATURE=',F7.2,' F  INITIAL/FUSION TEMPERATURE=',
     1F5.2,2H F,///,1H ,26X,'GEOMETRY',//,1H ,22X,'INNER RADI
     1S=',F10.5,3H FT,/,1H0,22X,'OUTER RADIUS=',F10.5,3H FT,/,1H0,20X
```

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```
     1,*WALL THICKNESS=*,F10.6,3H FT,///,1H ,17X,*THERMAL STORAGE MATER
     1IAL PROPERTIES*)
 350 FORMAT (1H0,27X,*DENSITY=*,F10.4," LBM/FT**3",/1H0,14X,*THERMAL CO
     1NDUCTIVITY=*,F10.6,* BTU/LBM-FT-F*,/,1HD,15X,*THERMAL DIFFUSIVITY=
     1*,F10.7," FT**2/HR",/,1H0,13X,*LATENT HEAT OF FUSION=*,F10.4,* BTU
     1/LBM*,///,1H ,20X,*CANNISTER MATERIAL PROPERTIES*,//,1H ,27X,*DENS
     1ITY=*,F10.4," LBM/FT**3",/,1H0,14X,*THERMAL CONDUCTIVITY=*,F10.6,*
     1 BTU/HR-FT-F*,/,1H0,15X,*THERMAL DIFFUSIVITY=*,F10.7," FT**2/HR")
 400 FORMAT (1 ///,1H ,3X,*TIME=*,F12.6,4X,*HEAT OUT=*,F9.6,5X,*SOLID V
     1OLUME=*,F9.6,/,1H ,15X,*ITERATIONS=*,I5,10X,*ERROR=*,E12.5,/,
     11H ,*INNER WALL TEMPERATURE=*,F9.6,7X,*OUTER WALL TEMPERATURE=*
     1,F9.6,/,1H ,23X,*INTERFACE RADIUS=*,F9.6,///,1H0,* RADIUS TEMP
     1 RADIUS TEMP RADIUS TEMP RADIUS TEMP*)
 500 FORMAT (13(1H ,8F8.4,/))
 450 FORMAT (///,1H ,10X,I4,* NODES RADIALLY*,13X,I4,* TIME STEPS*,//,1
     1H ,I4,* ITERATIONS ERROR=*,E7.1,6X,*RELAXATION FACTOR=*,F6.3)
     STOP
     END
```

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```
UD121,T30,IO15,CM60000,STCSA.F740579,BANDOW,KL121,229-2835
COMMENT.**********NO DECK**********
COMMENT.**********93392**********
ATTACH,F,ICE,CY=3.
FTN,I=F,L,R=3,B=FLGO.
RETURN,F.
REWIND,FLGO.
FLGO,PL=20000.
*EOR
     .035          4.11          181.          1310.          350.
    .0625        .1195833     8691.133      .7056356       .00001
     1.92        .2244971         13.5        501.12
    63   160   200    16     1     6     0
*EOR
*EOF
```

Figure C-2

Control Cards

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APPENDIX D

PROGRAM FIN

This appendix documents the program FIN used to calculate the temperature distribution and interface radius for the finned HP/TES systems. Figure D-1 is a program listing while Figure D-2 shows the control cards and sample input data. The input data is organized as follows:

Record 1, format 5E12.0

DIF - thermal diffusivity of salt, ft^2/hr
CON - conductivity of salt, BTU/ft-hr-^oF
DEN - density of salt, lbm/ft^3
TF - fusion temperature of salt, ^oF
HSL - latent heat of fusion of salt, BTU/lbm

Record 2, format 5E12.0

DFF - diffusivity of canister wall, ft^2/hr
CFN - conductivity of canister wall, BTU/ft-hr-^oF
DFN - density of canister wall, lbm/ft^3
QR - heat flux rate at outer edge of heat pipe, BTU/ft^2hr
DNR - ratio of solid salt volume to canister volume

Record 3, format 5E12.0

RI - outer radius of heat pipe wall, ft
RO - inner radius of outer canister wall, ft
B - half thickness of fin, ft
P - thickness of heat pipe wall, ft
G - length of fins, ft

Record 4, format 2E12.0

ORF - relaxation factor effecting speed of convergence
ERR - error tolerance on enthalpy change

Record 5, format 4I5

NR - number of nodes in radial direction

NT - number of points in time

NS - number of nodes in angular direction

NFN - number of fins

Record 6, format 4I5

IM - maximum number of iterations

IP - print every IPth time calculated

ID - print diagnostic if ID = 1

ICN - case number for identification

```
GRAM FIN         74/74   OPT=1                                FTN 4.6+446        0

      PROGRAM FIN(INPUT,TAPE1=INPUT,OUTPUT,TAPE2=OUTPUT)
      DIMENSION C1(25,30),C2(25,30),C3(25,30),C4(25,30),C5(25,30),
     1PH(25,30),T(25,30),H(25,30),NF(25),F(25),RF(30)
      READ (1,100) DIF,CON,DEN,TF,HSL
      READ (1,100) DFF,CFN,DFN,DF,DNR
      READ (1,100) RI,RO,B,P,G
      READ (1,100) OFF,ERR
      READ (1,200) NR,NT,NS,NFN
      READ (1,200) IM,IP,ID,ICN
      IC=IP
      IS=0
      JP=14
      RD=57.295779
      RMAX=RO/RI
      DR=(RO-RI)/RI/NR
      DS=3.1415927/NFN/NS
      ST=DR*RI/DIF/DEN/HSL
      DT=(RO+RO-RI+RI)*DNR/2./RI/RI/ST/NT
      E=DP*DR/DT
      FT=P/DT/PI
      TREF=G*RI/CON
      IF=INT(FT)
      IF (FT-AINT(FT).GT..5) IF=IF+1
      M=NR+2*IF
      M1=M-1
      N=NS
      N1=N-1
      DFF=DIF*CFN/DFF/CON
      DO 10 I=1,M
      NF(I)=NS+1
      F(I)=1.
      IF(I.LE.IF) GO TO 10
      IF(I.GT.M-IF) GO TO 10
      R=(2.*(I-IF)-1.)/2.*DR+1.
      A=B/RI/R/DS
      NF(I)=INT(A)
      F(I)=A-INT(A)
      IF(F(I).LT.(RI+G)/RI) GO TO 10
      NF(I)=0
      F(I)=0.
10    CONTINUE
      IF (ID.LT.1) GO TO 4
      WRITE (2,900) RMAX,DR,DS,ST,DT,B,FT,DFF
      WRITE (2,800) IC,IS,IF,M,M1,N,N1
      WRITE (2,700) (F(I),I=1,M)
      WRITE (2,900) (NF(I),I=1,M)
4     CONTINUE
      DO 20 I=1,M
      DO 20 J=1,M
```

Figure D-1. Program FIN

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```
      C1(I,J)=C2(I,J)=C3(I,J)=C4(I,J)=CON
      IF (J.LE.NF(I)) C1(I,J)=C2(I,J)=C3(I,J)=C4(I,J)=CFN
      IF (J.EQ.NF(I)+1) C1(I,J)=C3(I,J)=(1-F(I))*CON+F(I)*CFN
      IF (J.EQ.NF(I).AND.F(I).LT..5) C2(I,J)=CON*CFN/(.5*(CON+CFN)+F(I)
     1*(CON-CFN))
      IF (J.EQ.NF(I)+1.AND.F(I).LT..5) C4(I,J)=CON*CFN/(.5*(CON+CFN)+F(I
     1)*(CON-CFN))
      IF (J.EQ.NF(I)+1.AND.F(I).GE..5) C2(I,J)=CON*CFN/(.5*(3.*CFN-CON)+
     1F(I)*(CON-CFN))
      IF (J.EQ.NF(I)+1.AND.F(I).GE..5) C4(I,J)=CFN
      IF (J.EQ.NF(I)+2.AND.F(I).GE..5) C4(I,J)=CON*CFN/(.5*(3.*CFN-CON)+
     1F(I)*(CON-CFN))
20    CONTINUE
      IF (ID.LT.1) GO TO 6
      WRITE (2,700) ((C1(I,J),J=1,N),I=1,M)
      WRITE (2,700) ((C2(I,J),J=1,N),I=1,M)
      WRITE (2,700) ((C3(I,J),J=1,N),I=1,M)
      WRITE (2,700) ((C4(I,J),J=1,N),I=1,M)
6     CONTINUE
      DO 30 I=1,M
      R=1.+(2.*(I-IF)-1.)/2.*DR
      DO 30 J=1,N
      IF (I.EQ.M) GO TO 40
      C1(I,J)=2.*C1(I,J)*C3(I+1,J)/(C1(I,J)+C3(I+1,J))
      C3(I+1,J)=C1(I,J)
40    C1(I,J)=C1(I,J)/CON*(1.+DR/2./R)
      C2(I,J)=C2(I,J)*(DR/R/DS)**2./CON
      C3(I,J)=C3(I,J)/CON*(1.-DR/2./R)
      C4(I,J)=C4(I,J)*(DS/R/DS)**2./CON
      IF (I.EQ.1) C3(I,J)=0.
      IF (J.EQ.1) C4(I,J)=0.
      IF (I.EQ.M) C1(I,J)=0.
      IF (J.EQ.N) C2(I,J)=0.
      FF=0.
      IF (J.LE.NF(I)) FF=1.
      IF (J.EQ.NF(I)+1) FF=F(I)
      C5(I,J)=0.+(C1(I,J)+C2(I,J)+C3(I,J)+C4(I,J))/(1.-FF*(1.-DFR))
      T(I,J)=0.
      PH(I,J)=1.
      IF (J.LE.NF(I)) PH(I,J)=0.
      IF (J.EQ.NF(I)+1) PH(I,J)=1.-F(I)
      H(I,J)=PH(I,J)
30    CONTINUE
      IF (ID.LT.1) GO TO 7
      WRITE (2,700) ((C1(I,J),C2(I,J),C3(I,J),C4(I,J),C5(I,J),
     1J=1,N),I=1,M)
      WRITE (2,700) ((H(I,J),J=1,N),I=1,M)
      WRITE (2,700) ((PH(I,J),J=1,N),I=1,M)
      WRITE (2,700) ((T(I,J),J=1,N),I=1,M)
7     CONTINUE
      DNS=DNS*100
      WRITE (2,300)ICN,NFN,QR,DNS,F,NFN,RI,B,RO,G
      DNS=DNS/100
      WRITE (2,350)DEN,DFN,CON,CFN,DIF,DFF,HSL,M,N,NT,ORF,ERF,IM
      WRITE (2,360)TREF,TF,ST
```


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```
60      IT=0
95      DO 6 J=1,N
8       RF(J)=0.
        OH=H(1,1)
        H(1,1)=(D*PH(1,1)+C1(1,1)*T(2,1)+C2(1,1)*T(1,2)-DR*ST)/C5(1,1)
        H(1,1)=ORF*(H(1,1)-OH)+OH
        T(1,1)=H(1,1)/DFR
        EN=ABS(H(1,1)-OH)
        ED=ABS(H(1,1))
        DO 50 I=2,M1
        OH=H(I,1)
        H(I,1)=(D*PH(I,1)+C1(I,1)*T(I+1,1)+C2(I,1)*T(I,2)+C3(I,1)*T(I-1,1
       1))/C5(I,1)
        IF (H(I,1).LT.0.) GO TO 60
        H(I,1)=H(I,1)*C5(I,1)/D
        T(I,1)=0.
        IF (RF(1).LT.1.) RF(1)=(I-IF-H(I,J))*DR+1.
        GO TO 50
60      IF (OH.LT.0.) H(I,1)=ORF*(H(I,1)-OH)+OH
        FF=F(I)
        IF (NF(I).GT.0) FF=1.
        T(I,1)=H(I,1)/(1.-FF*(1.-DFR))
        EN=EN+ABS(H(I,1)-OH)
        ED=ED+ABS(H(I,1))
50      CONTINUE
        OH=H(M,1)
        H(M,1)=(D*PH(M,1)+C2(M,1)*T(M,2)+C3(M,1)*T(M-1,1))/C5(M,1)
        IF(H(M,1).LT.0) GO TO 90
        H(M,1)=H(M,1)*C5(M,1)/D
        T(M,1)=0.
        GO TO 70
90      IF(OH.LT.0.) H(M,1)=ORF*(H(M,1)-OH)+OH
        T(M,1)=H(M,1)/DFR
        EN=EN+ABS(H(M,1)-OH)
        ED=ED+ABS(H(M,1))
70      CONTINUE
        DO 5 J=2,N1
        OH=H(1,J)
        H(1,J)=(D*PH(1,J)+C1(1,J)*T(2,J)+C2(1,J)*T(1,J+1)+C4(1,J)*T(1,J-1
       1-D.*ST)/C5(1,J)
        H(1,J)=ORF*(H(1,J)-OH)+OH
        T(1,J)=H(1,J)/DFR
        EN=EN+ABS(H(1,J)-OH)
        ED=ED+ABS(H(1,J))
        DO 15 I=2,M1
        OH=H(I,J)
        H(I,J)=(D*PH(I,J)+C1(I,J)*T(I+1,J)+C2(I,J)*T(I,J+1)
       1+C3(I,J)*T(I-1,J)+C4(I,J)*T(I,J-1))/C5(I,J)
        IF (H(I,J).LT.0) GO TO 25
        H(I,J)=H(I,J)*C5(I,J)/D
        T(I,J)=0.
        IF (RF(J).LT.1.) RF(J)=(I-IF-H(I,J))*DR+1.
        GO TO 15
```


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```
25      IF(OH.LT.0.) H(I,J)=(H(I,J)-OH)*ORF+OH
        FF=0.
        IF (J.LE.NF(I)) FF=1.
        IF (NF(I)+1.EQ.J) FF=F(I)
        T(I,J)=H(I,J)/(1.-FF*(1.-DFR))
        EN=EN+ABS(H(I,J)-OH)
        ED=ED+ABS(H(I,J))
15      CONTINUE
        OH=H(M,J)
        H(M,J)=(D*PH(M,J)+C2(M,J)*T(M,J+1)+C3(M,J)*T(M-1,J)
       1+C4(M,J)*T(M,J-1))/C5(M,J)
        IF (H(M,J).LT.0.) GO TO 45
        H(M,J)=H(M,J)*C5(M,J)/D
        T(M,J)=0.

        GO TO 35
45      IF (OH.LT.0.) H(M,J)=ORF*(H(M,J)-OH)+OH
        T(M,J)=H(M,J)/DFR
        EN=EN+ABS(H(M,J)-OH)
        ED=ED+ABS(H(M,J))
35      CONTINUE
5       CONTINUE
        OH=H(1,N)
        H(1,N)=(D*PH(1,N)+C1(1,N)*T(2,N)+C4(1,N)*T(1,N-1)
       1-D*ST)/C5(1,N)
        H(1,N)=ORF*(H(1,N)-OH)+OH
        T(1,N)=H(1,N)/DFR
        EN=EN+ABS(H(1,N)-OH)
        ED=ED+ABS(H(1,N))
        DO 55 I=2,M1
        OH=H(I,N)
        H(I,N)=(D*PH(I,N)+C1(I,N)*T(I+1,N)+C3(I,N)*T(I-1,N)
       1+C4(I,N)*T(I,N-1))/C5(I,N)
        IF (H(I,N).LT.0.) GO TO 65
        H(I,N)=H(I,N)*C5(I,N)/D
        T(I,N)=0.
        IF (RF(N).LT.1.) RF(N)=(I-IF-H(I,N))*DF+1.
        GO TO 55
65      IF (OH.LT.0.) H(I,N)=ORF*(H(I,N)-OH)+OH
        FF=0.
        IF (NF(I).GE.N) FF=1.
        T(I,N)=H(I,N)/(1.-FF*(1.-DFR))
        EN=EN+ABS(H(I,N)-OH)
        ED=ED+ABS(H(I,N))
55      CONTINUE
        OH=H(M,N)
        H(M,N)=(D*PH(M,N)+C3(M,N)*T(M-1,N)+C4(M,N)*T(M,N-1))/C5(M,N)
        T(M,N)=H(M,N)/DFR
75      CONTINUE
        IF (EN.LT.ED*ERR) GO TO 85
        IT=IT+1
        IF(IT.LT.IM) GO TO 35
85      IS=IS+1
        IC=IC+1
        IF (IC.LT.IP) GO TO 2
        EN=0.
```

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```
      HE=0.
      DO 1 I=1,M
      R=1.+(I-IF-.5)*DR
      DO 1 J=1,N
      HI=1.
      IF (J.LE.NF(I)) HI=0.
      IF (J.EQ.NF(I)+1) HI=F(I)
      HJ=AMAX1(0.,H(I,J))
      SM=SM+R*DS*DR*(HI-HJ)
      HE=HE+R*DS*DR*(HI-H(I,J))
1     PH(I,J)=T(I,J)/ST
      ER=EN/ED
      TP=DT*IS
      SM=SM*2.*NFN/3.1415927
      HE=HE*2.*NFN/3.1415927
      RE=SQRT(1.+SM)

      WRITE (2,400) TP,RE,HE,SM,IT,ER
      J1=1
13    J2=J1+JP
      J2=AMIN0(J2,N)
      WRITE (2,450) (RF(J),J=J1,J2)
      DO 9 J=J1,J2
9     RF(J)=(J-.5)*DS*RD
      WRITE(2,500) (RF(J),J=J1,J2)
      DO 11 J=J1,J2
11    RF(J)=(3.*T(1,J)-T(2,J))/2./ST
      WRITE (2,550) (RF(J),J=J1,J2)
      DO 14 J=J1,J2
14    RF(J)=(CON*T(IF+1,J)+CFN*T(IF,J))/(CON+CFN)/ST
      WRITE (2,750) (RF(J),J=J1,J2)
      WRITE (2,600)
      DO 12 I=1,M
      R=1.+(I-IF-.5)*DR
12    WRITE (2,650) R,(PH(I,J),J=J1,J2)
      J1=J2+1
      IF (J1.LE.N) GO TO 13
      IC=0
      IF (IS.LE.1) IC=1
2     DO 3 I=1,M
      DO 3 J=1,N
3     PH(I,J)=H(I,J)
      IF (IS.LT.MT) GO TO 80
100   FORMAT (5E12.0)
200   FORMAT (5I5)
300   FORMAT (1H1,/,2(1H0,/),1H ,34X,*HEAT TRANSFER IN PHASE CHANGE/THER
     1MAL ENERGY STORAGE SYSTEM*,/,1H0,36X,*CASE NUMBER*,I3,*  DIMENS
     1IONLESS OUTPUT*,I10,* FINS*,/,1H0,25X,"HEAT FLUX RATE (BTU/FT**2/HR
     1)=",F10.3,11X,*FILLED VOLUME=*,F5.1,* PERCENT*,///,1H0,37X,* CONV
     1STER GEOMETRY*,22X,*FIN GEOMETRY*,/,1H0,32X,*WALL THICKNESS,FT=*,F
     110.8,12X,*NUMBER OF FINS=*,I3,/,1H0,34X,*INNER RADIUS,FT=*,F10.5,
     10X,*FIN THICKNESS,FT=*,F10.8,/,1H0,34X,*OUTER RADIUS,FT=*,F10.8,1.
```

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```
      1X,*FIN LENGTH,FT=*,F10.6)
350   FORMAT (///,1H ,56X,* MATERIAL PROPERTIES*,//,1H0,39X,*PROPERTY*
     1,20X,*PHASE CHANGE MATERIAL*,4X,*CANNISTER MATERIAL*,/,1H0,27X,
     1"DENSITY,LBM/FT**3",30X,F8.3,15X,F8.3,/,1H0,27X,*THERMAL CONDUCTI
     1IVITY,BTU/FT/HR/F*,15X,F8.5,15X,F8.4,/,1H0,27X,"THERMAL DIFFUSIVITY
     1FT**2/HR",25X,F8.6,15X,F8.6,/,1H0,27X,*LATENT HEAT OF FUSION,BTU/
     1BM*,12X,F8.3,///,1H ,52X,*ITERATION CONTROL PARAMETERS*,//,1H ,5X
     1*NODAL STRUCTURE:*,I5,* NODES RADIALLY*,I5,* ANGULAR NODES*,I5,*
     1IME STEPS*,/,1H0,5X,*ITERATION PARAMETERS:   OVER-RELAXATION FAC
     10R=*,F6.3,5X,*ERROR TOLERANCE=*,E12.4,*   MAXIMUM ITERATIONS=*,I5
360   FORMAT (///,1H ,5X,*REFERENCE TEMPERATURE,F=*,F8.3,11X,*FUSION/IN
     1ITIAL TEMPERATURE,F=*,F9.3,12X,*STEFAN NUMBER=*,F12.3)
400   FORMAT (1H1,*TIME=*,F8.4,3X,*EFF RADIUS=*,F9.6,3X,*HEAT EXTRACTED=
     1*,F9.6,3X,*SOLID VOLUME=*,F8.6,3X,*ITERATIONS=*,I4,3X,
     1*ERROR=*,E12.5)
450   FORMAT (1H0,*  S/L RADIUS*,15F8.4)
500   FORMAT (1H ,*FIN OC ANGLE*,15F8.1)
550   FORMAT (1H ,*ID WALL TEMP*,15F8.4)
600   FORMAT (1H ,6X,*RADIUS*,20X,*NODAL DIMENSIONLESS TEMPERATURES   (
     1-TF)/TR*)
650   FORMAT (1H ,4X,16F8.4)
700   FORMAT (1H ,15F8.4)
750   FORMAT (1H ,*OD WALL TEMP*,15F8.4)
900   FORMAT (1H ,10E12.5)
900   FORMAT (1H ,15I5)
      STOP
      END
```

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```
UD121,T200,IO30,CM60000,STCSA.P740579,BANDOW,KL121,229-2835
COMMENT.**********NO DECK**********
COMMENT.**********93392**********
ATTACH,F,FIN,CY=1.
FTN,I=F,L,R=3,B=FLGO.
RETURN,F.
REWIND,FLGO.
FLGO,FL=20000.
*EOR
     .035          4.11          181.          1310.          350.
   .2244971        13.5         501.12        8691.133      .7056356
    .0625         .1195833    .002604167   .005416667    .05447917
     1.93          .00001
    21    80    30     6
   200     8     0    20
*EOR
*EOF
```

Figure D-2

Control Cards

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